CBCS SCHEME

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21MATCS41

Fourth Semester B.E. Degree Examination, June/July 2024
Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs. Max. Marks: 100

Note: **1.** ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. ***Provide data table book.***

Module-1

1 a. Define tautology. Show that the compound proposition $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology for any propositions p, q, r. **(06 Marks)**

b. Prove that (i) $p \vee [p \wedge (p \vee q)] \equiv p$ (ii) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \equiv p \wedge q$ using the laws of logic. **(07 Marks)**

c. Prove that for all integers k and ℓ is k and ℓ are both odd, then $k + \ell$ is even and $k\ell$ is odd. **(07 Marks)**

OR

2 a. Define: (i) Universal quantifiers (ii) Existential quantifiers, with an example. **(06 Marks)**

b. Test the validity of the following argument.
I will become famous or I will not become a musician.
I will become a musician.
Therefore I will become famous. **(07 Marks)**

c. Suppose the universe consist of integers. Consider the following open statements:
$p(x) : x \leq 3$, $q(x) : x + 1$ is odd $r(x) : x > 0$.
Write down the truth values of:
(i) $p(2)$ (ii) $\vee q(4)$ (iii) $p(-1) \wedge q(1)$ (iv) $\sim p(3) \vee r(0)$
(v) $p(0) \rightarrow q(0)$ (vi) $p(1) \leftrightarrow \sim q(2)$ (vii) $p(4) \vee (q(1) \wedge r(2))$ **(07 Marks)**

Module-2

3 a. Let A and B be finite sets with $|A| = m$ and $|B| = n$. Find how many one to one functions are possible from A to B. If there are 60 1 – 1 functions from A to B and $|A| = 3$, what is $|B|$? **(06 Marks)**

b. Let $A = \{1, 2, 3, 4, 6, 12\}$ and R be a relation on A defined by aRb if "a is a multiple of b". Write down the relation R, relation matrix M(R) and draw its digraph. **(07 Marks)**

c. Define: (i) Null graph (ii) Bipartite graph (iii) Euler circuit. Give an example for each. **(07 Marks)**

OR

4 a. Draw the Hasse diagram representing the positive divisors of 48. **(06 Marks)**

b. Consider the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1 \ \forall \ x \in R$. Find gof, fog, f^2. **(07 Marks)**

c. Define isomorphism of graphs. Prove that 2 graphs below are isomorphic.

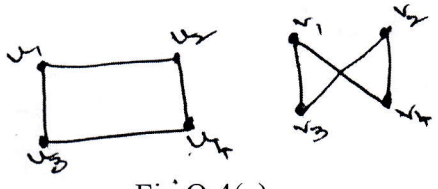

Fig.Q.4(c) **(07 Marks)**

Module-3

5 a. Find the correlation coefficient between the speed and the stopping distance and the equations of regression lines.

Speed, x	16	24	32	40	48	56
Stopping distance, y	0.39	0.75	1.23	1.91	2.77	3.81

(06 Marks)

b. Fit a best curve of the form $y = ax^b$ for the following data:

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

c. Fit a straight line by the method of least squares.

x	1	2	3	4	5
y	14	13	9	5	2

(07 Marks)

OR

6 a. The following are the percentage of marks in 2 subjects of 9 students. Find the rank correlation coefficient.

x	38	50	42	61	43	55	67	46	72
y	41	64	70	75	44	55	62	56	60

(06 Marks)

b. Fit a 2nd degree parabola $y = a + bx + cx^2$ for the data:

x	0	1	2	3	4	5
y	1	3	7	13	21	31

(07 Marks)

c. Given that $8x - 10y + 66 = 0$ and $40x - 18y = 214$ are the regression equations. Find the means of x and y and correlation coefficient. Find σ_y if $\sigma_x = 3$. **(07 Marks)**

Module-4

7 a. A random variable X has the following probability function:

x	–2	–1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

Find: (i) K (ii) $P(X < 1)$ (iii) $P(X > -1)$ **(06 Marks)**

b. Find the mean and standard deviation of Poisson distribution. **(07 Marks)**

c. The mean weight of 500 students in a school is 50 kgs and the standard deviation is 6 kgs. Assuming that the weights are normally distributed, find the expected number of students weighing (i) between 40 and 50 kg (ii) more than 60 kg. Given that A(1.67) = 0.4525. **(07 Marks)**

OR

8 a. Find the constant K such that

$$f(x) = \begin{cases} Kx^2, & 0 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function. Find the mean. **(06 Marks)**

b. When an honest coin is tossed 4 times, find the probability of getting:
(i) exactly one head (ii) atmost 3 heads (iii) at least 2 heads **(07 Marks)**

c. The probability that an individual suffers a bad reaction from a certain injection is 0.001. Using Poisson distribution, find the probability that out of 2000 individuals:
(i) exactly 3 (ii) more than 2 will suffer a bad reaction. **(07 Marks)**

Module-5

9 a. X and Y are independent random variables such that X takes 1, 5 with probabilities $\frac{1}{2}$, $\frac{1}{2}$ respectively. Y takes –4, 2, 7 with probabilities $\frac{3}{8}$, $\frac{3}{8}$ and $\frac{1}{4}$ respectively. Find the joint probability distribution of X and Y. Find Cov (X, Y). **(06 Marks)**

b. Find the student 't' for the following variables values in a sample of eight –4, –2, –2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. **(07 Marks)**

c. The following are the I.Q's of a randomly chosen sample of 10 boys: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this data support the hypothesis that the population mean of I.Q's is 100 at 5% level of significance? **(07 Marks)**

OR

10 a. Explain the terms:
(i) Null hypothesis
(ii) Alternate hypothesis
(iii) Levels of significance
(iv) Type 1 and Type 2 errors **(06 Marks)**

b. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table:

x	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased. Use Chisquare test at 5% level of significantly. **(07 Marks)**

c. The nine items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5 ($t_{0.05} = 2.31$). **(07 Marks)**

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