CBCS SCHEME

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21MATME41

Fourth Semester B.E. Degree Examination, June/July 2024
Complex Analysis, Probability and Linear Programming

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that $w = f(z) = \log z$ $(z \neq 0)$ is analytic, using Cauchy – Riemann equation and find $\frac{dw}{dz}$ **(06 Marks)**

b. Derive Cauchy – Riemann equation in Cartesian form. **(07 Marks)**

c. Find the analytic function f(z) whose imaginary part is $e^x(x \sin y + y \cos y)$ **(07 Marks)**

OR

2 a. Show that f(z) = coshz is analytic and hence find f'(z). **(06 Marks)**

b. If f(z) is analytic function show that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$ **(07 Marks)**

c. Find the analytic functions whose real part is $\frac{x^4 - y^4 - 2x}{x^2 + y^2}$. Hence determine V. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = e^z$. **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, –1 in to w = 2, i, –2. **(07 Marks)**

OR

4 a. Find the bilinear transformation which maps the points $z = \infty$, i, 0 into w = –1, –i, 1. **(06 Marks)**

b. Discuss the transformation $w = z + \frac{1}{z}$ **(07 Marks)**

c. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle (i) |z| = 3, (ii) $|z| = \frac{1}{2}$ **(07 Marks)**

Module-3

5 a. The probability density function of a variate X is given by the following table.

x	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

Find K. Also find $P(x \geq 5)$ and $P(3 < x \leq 6)$ **(06 Marks)**

b. Find the Mean and Variance of a Poisson distribution. **(07 Marks)**

c. The number of telephone lines busy at an instant of time is binomial variate with probability 0.1 that a line is busy if 10 lines are choosen at random, what is the probability that (i) no line is busy (ii) all lines are busy (iii) atleast one line is busy (iv) atmost 2 lines are busy. **(07 Marks)**

OR

6 a. The probability density function of a random variable X is

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) the value of K, (ii) $P(1 < x < 2)$, (iii) $P(x \leq 1)$ **(06 Marks)**

b. Find the mean and variance of binomial distribution. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviations. Find the number of students whose marks will be
(i) Less than 65, (ii) More than 75 (iii) Between 65 and 75. **(07 Marks)**

Module-4

7 a. Using the Simplex method to solve the L.P.P.
Maximize $Z = 5x_1 + 7x_2$
Subject to constraint $x_1 + x_2 \leq 4$
$3x_1 - 8x_2 \leq 24$
$10x_1 + 7x_2 \leq 35$
and $x_1, x_2 \geq 0$ **(10 Marks)**

b. Use Big-M method to solve the L.P.P.
Maximize $Z = -2x_1 - x_2$
Subject to constraint $3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 3$
$x_1 + 2x_2 \leq 4$
and $x_1, x_2 \geq 0$ **(10 Marks)**

OR

8 a. Define the following terms :
i) A linear Programming problems
ii) Basic solution
iii) Basic feasible solution
iv) Optional solution
v) Artificial variables of an LPP. **(10 Marks)**

b. Use Big-M method to solve the LPP.
Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$
Subject to constraints $x_1 + 2x_2 + 3x_3 = 15$
$2x_1 + x_2 + 5x_3 = 20$
$x_1 + 2x_2 + x_3 + x_4 = 10$
$x_1, x_2, x_3, x_4 \geq 0$ **(10 Marks)**

Module-5

9 a. Find the feasible solution to the following transportation problem using North West corner method.

	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
	6	10	15	4	

(10 Marks)

b. The processing time in hours for the Jobs when allocated to the different machines are indicated below. Assign the machines for the Jobs so that the total processing time is minimum.

		Machines				
		M_1	M_2	M_3	M_4	M_5
Jobs	J_1	9	22	58	11	19
	J_2	43	78	72	50	63
	J_3	41	28	91	37	45
	J_4	74	42	27	49	39
	J_5	36	11	57	22	25

(10 Marks)

OR

10 a. Solve the following transportation problem by least cost method.

5	4	3	6
4	7	6	8
2	5	8	12
8	6	7	4
8	10	12	

(10 Marks)

b. Four jobs are to be done on four different machines. The cost (in rupees) of producing i^{th} Job on the J^{th} machine is given below.

		Machines			
	J \ M	M_1	M_2	M_3	M_4
Jobs	J_1	15	11	13	15
	J_2	17	12	12	13
	J_3	14	15	10	14
	J_4	16	13	11	17

Assign the Jobs to different machines so as to minimize the total cost. **(10 Marks)**

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