CBCS SCHEME

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21MAT31

Third Semester B.E. Degree Examination, June/July 2024
Transform Calculus, Fourier Series & Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace Transform of, $\left(\frac{4t+5}{e^{2t}}\right)^2$. **(06 Marks)**

b. The square wave function f(t) with period 2a is defined by,
$f(t) = t\ ;\ 0 \le t \le a$
$= 2a - t\ ;\ a \le t \le 2a$
Find L[f(t)]. **(07 Marks)**

c. Evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$ by applying convolution theorem. **(07 Marks)**

OR

2 a. Find inverse Laplace transform $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. **(06 Marks)**

b. Express the following function in terms of unit step function and hence find the Laplace transform.
$f(t) = 1;\ 0 < t \le 1$
$= t;\ 1 \le t \le 2$
$= t^2;\ t > 2$. **(07 Marks)**

c. Applying Laplace transform, solve the differential equation,
$y''(t) + 4y'(t) + 4y(t) = e^{-t}$,
Subject to the condition y(0) = y′(0) = 0. **(07 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x) = x^2$ over the interval $[-\pi, \pi]$, hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots + \infty$. **(06 Marks)**

b. Obtain the half range sine series of the function, $f(x) = x$ in the interval (0, 2). **(07 Marks)**

c. Obtain the constant term and co-efficient of first cosine and sine terms in the expansion of y from the following table :

x	0°	60°	120°	180°	240°	300°	360°
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

(07 Marks)

OR

4 a. Find the Fourier series of $f(x) = 2 - x\ ;\ 0 \le x \le 4$
$x - 6\ ;\ 4 \le x \le 8$ **(06 Marks)**

b. Obtain the half range sine series of the function, $f(x) = x^2$ over $(0, \pi)$. **(07 Marks)**

c. Obtain a_0, a_1, b_1 in the Fourier expansion of y using harmonic analysis for the data given,

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

Module-3

5 a. Find the Fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$; $\alpha > 0$. **(06 Marks)**

b. Obtain the inverse z-transform of, $\dfrac{2z^2 + 3z}{(z^2 - 2z - 8)}$. **(07 Marks)**

c. Find the Fourier transform of,

$$f(x) = x^2; \ |x| < a$$
$$= 0; |x| > a$$

where a is +ve constant. **(07 Marks)**

OR

6 a. Find the Complex Fourier transform of the function,

$$f(x) = 1 \quad \text{for } |x| \le a$$
$$= 0 \quad \text{for } |x| > a$$

Hence deduce, evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Evaluate $Z_T\left[2n + \sin\left(\frac{n\pi}{4}\right) + 1\right]$. **(07 Marks)**

c. Solve the difference equation, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-Transform. **(07 Marks)**

Module-4

7 a. Classify the following partial differential equation,

(i) $\dfrac{\partial^2 u}{\partial x^2} + 4\dfrac{\partial^2 u}{\partial x \partial y} + 4\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial x} + 2\dfrac{\partial u}{\partial y} = 0$.

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + (1 - y^2)\dfrac{\partial^2 u}{\partial y^2} = 0, \ -\infty < x < \infty, \ -1 < y < 1$.

(iii) $(1 + x^2)\dfrac{\partial^2 u}{\partial x^2} + (5 + 2x^2)\dfrac{\partial^2 u}{\partial x \partial t} + (4 + x^2)\dfrac{\partial^2 u}{\partial t^2} = 0$

(iv) $(x + 1)\dfrac{\partial^2 u}{\partial x^2} - 2(x + 2)\dfrac{\partial^2 u}{\partial x \partial y} + (x + 3)\dfrac{\partial^2 u}{\partial y^2} = 0$ **(10 Marks)**

b. Find the numerical solution of the parabolic equation $\dfrac{\partial^2 u}{\partial x^2} = 2\dfrac{\partial u}{\partial t}$, using Schmidt formula. Given $u(0,t) = 0 = u(4,t)$ and $u(x, 0) = x(4 - x)$ by taking h = 1 find the values upto t = 5. **(10 Marks)**

OR

8 a. Solve $u_{xx} + u_{yy} = 0$ in the following square region with the boundary conditions as indicated in the Fig. Q8 (a). **(10 Marks)**

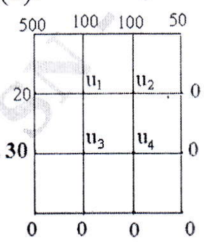

Fig. Q8 (a)

b. Solve numerically $u_{xx} = 0.0625\, u_{tt}$, subject to the conditions u(0, t) = 0 = u(5, t), $u(x, 0) = x^2(x - 5)$ and $u_t(x,0) = 0$ by taking h = 1 for $0 \le t \le 1$. **(10 Marks)**

Module-5

9 a. Use Runge-Kutta method to find y(0.2) for the equation, $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$. Given that $y = 1,\ y' = 0$ when $x = 0$. **(06 Marks)**

b. Find the curves on which the function, $\int_0^1 \{(y')^2 + 12xy\}dx$ with y(0) = 0 and y(1) = 1 can be extremised. **(07 Marks)**

c. Derive the Eulers equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ **(07 Marks)**

OR

10 a. Solve the differential equation $y'' + xy' + y = 0$ for x = 0.4, using Milne's predictor-corrector formula given that, **(06 Marks)**

x	0	0.1	0.2	0.3
y	1	0.995	0.9802	0.956
$\frac{dy}{dx}$	0	-0.0995	-0.196	-0.2863

b. Find the curve on which functional $\int_0^{\frac{\pi}{2}} [(y')^2 - y^2 + 2xy]dx$ with $y(0) = y\left(\frac{\pi}{2}\right) = 0$ can be extremized. **(07 Marks)**

c. Prove that shortest distance between two points in a plane is a straight line. **(07 Marks)**

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