

CBCS SCHEME

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21MATCS41

Fourth Semester B.E. Degree Examination, June/July 2023 Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define tautology. Determine whether the following compound statement is a tautology or not: $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$ (06 Marks)
- b. Determine whether the following argument is valid or not : (07 Marks)
No engineering student of I & II semester studies logic
Anil is an engineering student who studies logic
 \therefore Anil is not in II semester.
- c. Give direct proof and proof by contradiction for the statement "If n is an odd integer then $n + 9$ is an even integer." (07 Marks)

OR

- 2 a. Prove that the argument given below is a valid argument: (06 Marks)

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \sim q \rightarrow \sim p \\ \hline p \\ \hline \therefore r \end{array}$$
- b. Prove that $[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \leftrightarrow r$ by using laws of logic. (07 Marks)
- c. Give a direct proof for each of the following, For all integers K and ℓ , if K and ℓ are both even, then (i) $K + \ell$ is even (ii) $K\ell$ is even (07 Marks)

Module-2

- 3 a. Consider $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 5$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f . (06 Marks)
- b. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b ". Write down the relation R , relation matrix $M(R)$ and draw its diagram. (07 Marks)
- c. Define: (i) Simple graph (ii) Complete graph (iii) Subgraph (iv) Spanning subgroup (07 Marks)

OR

- 4 a. Let f and g be functions from \mathbb{R} to \mathbb{R} defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$, if $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a and b . (06 Marks)
- b. Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$ be a relation on A . Determine whether the relation R is reflexive, symmetric, anti-symmetric or transitive. Hence verify R is an equivalence relation or not. (07 Marks)

- c. Define isomorphism of graph. Find whether the following graphs are isomorphic or not.

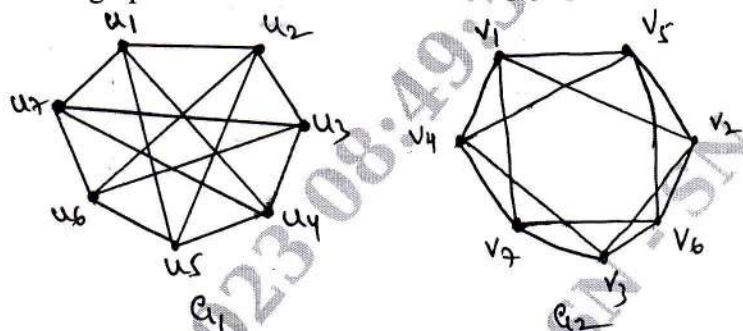


Fig.Q4(c)

(07 Marks)

Module-3

- 5 a. Fit a second degree parabola $y = a + bx + cx^2$ in the least square sense for the following data:

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

(06 Marks)

- b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

- c. The following are the percentage of marks in matrix (x) and statistics (y) of nine students. Calculate the rank correlation coefficient.

x	38	50	42	61	43	55	67	46	72
y	41	64	70	75	44	55	62	56	60

(07 Marks)

OR

- 6 a. Fit a least square geometric curve $y = ax^b$ for the data:

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(06 Marks)

- b. Given the equation of the regression lines $x = 19.13 - 0.874y$, $y = 11.64 - 0.5x$. Compute the mean of x, y and the coefficient of correlation.

(07 Marks)

- c. Three judges A, B, C, give the following ranks. Find which pair of judges has common approach.

A	1	6	5	10	3	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	5	7

(07 Marks)

Module-4

- 7 a. A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	3K	3K	K ²	2K ²	7K ² +K

- (i) Find K (ii) Find $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$

(06 Marks)

- b. Derive the mean and standard deviation of binomial distribution.

(07 Marks)

- c. In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and SD of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2100 hours (ii) less than 1950 hours (iii) between 1900 to 2100 hours. $A(1.67) = 0.4525$, $A(0.83) = 0.2967$, $A(1.67) = 0.4525$. (07 Marks)

OR

- 8 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer? (06 Marks)
- b. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with :
(i) no accident in a year (ii) more than 3 accidents in a year. (07 Marks)
- c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D. of the distribution. $A(0.5) = 0.1915$, $A(1.4) = 0.4192$. (07 Marks)

Module-5

- 9 a. The joint distribution of two random variables x and y as follows:

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
2	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

- Find: $E(X)$, $E(Y)$ and $E(XY)$ (ii) $\text{cov}(X, Y)$ (iii) $\rho(X, Y)$ (06 Marks)
- b. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased one. (07 Marks)
- c. A certain stimuluX administered to each of the 12 patients resulted in the following changes of blood pressure. 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure, ($t_{0.05} = 2.201$ for 11 d.f) (07 Marks)

OR

- 10 a. A fair coin is tossed thrice. The random variables X and Y are defined as follows:
 $X = 0$ or 1 according as head or tail occurs in first toss.
 $Y =$ Number of heads.
(i) Determine the distribution of X and Y .
(ii) Determine the joint distribution of X and Y . (06 Marks)
- b. Explain: (i) Null hypothesis (ii) Significance level (iii) Type I and Type II Errors (07 Marks)
- c. Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below :

Number of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution. ($\chi^2_{0.05} = 11.07$ for 5 d.f.)

(07 Marks)
