CBCS SCHEME

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21MAT11

First Semester B.E. Degree Examination, July/August 2022
Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. With usual notation prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$. **(06 Marks)**

b. Find the angle between the curves $r = 2\sin\theta$ and $r = 2\cos\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$. **(07 Marks)**

OR

2 a. With usual notation prove that

$$\rho = \frac{\left(1+y_1^2\right)^{\frac{3}{2}}}{y_2}.$$ **(06 Marks)**

b. Find the radius of curvature for the curve $r^n = a^n \sin n\theta$. **(07 Marks)**

c. Show that $r = 4\sec^2\theta/2$ and $r = 9\operatorname{cosec}^2\theta/2$ the pair of curves cut orthogonally. **(07 Marks)**

Module-2

3 a. Expand $Y = \log(1+\sin x)$ upto the term contains x^4 by Maclaurin's series expansion. **(06 Marks)**

b. If $u = f(2x-3y, 3y-4z, 4z-2x)$, show that $6u_x + 4u_y + 3u_z = 0$. **(07 Marks)**

c. Show that the function $f(x, y) = x^3 + y^3 - 63x - 63y + 12xy$ is maximum at $(-7, -7)$. **(07 Marks)**

OR

4 a. Evaluate $\underset{x\to 0}{\text{Lt}}\,(\cos x)^{\frac{1}{x^2}}$. **(06 Marks)**

b. If $u = x^3 + y^3$ where $x = a\cos t$, $y = b\sin t$ find $\frac{dU}{dt}$. **(07 Marks)**

c. If $U = e^x\cos y$, $V = e^x \sin y$. Find $\frac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

Module-3

5 a. Solve for P : $xp^2 + (y - x)p - y = 0$; where $p = \frac{dy}{dx}$. **(06 Marks)**

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

c. Solve $(x^2 + y^2 + x)\,dx + xy\,dy = 0$. **(07 Marks)**

OR

6 a. Solve : $[1 + \log x + \log y]dx + \left(1 + \frac{x}{y}\right)dy = 0$. **(06 Marks)**

b. Solve : $p = \sin(y - xp)$. Also find its singular solutions. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Module-4

7 a. Solve : $\frac{d^4y}{dx^4} - 4\frac{d^2y}{dx^2} + 4y = 0$. **(06 Marks)**

b. Solve : $(D^2 + 4)y = e^x + \cos 2x$. **(07 Marks)**

c. Using Variation of parameter method, solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x.$$ **(07 Marks)**

OR

8 a. Solve : $(D^2 - 1)y = 1 + x + x^2$. **(06 Marks)**

b. Solve : $(D^2 + D + 1)y = (1 - e^x)$. **(07 Marks)**

c. Solve : $(1 + x)^2 \frac{dy^2}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2\sin[\log(1 + x)]$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix :

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}.$$

(06 Marks)

b. Find for what values of λ and μ the system of linear equations :

$x + y + z = 6$
$x + 2y + 5z = 10$
$2x + 3y + \lambda z = \mu$

has

i) a unique solution
ii) no solution
iii) Infinitely many solutions. **(07 Marks)**

c. Solve the system of equations :

$10x + 2y + z = 9$
$2x + 20y - 2z = -44$
$-2x + 3y + 10z = 22$

by Gauss Seidel method taking (0, 0, 0) as an initial approximate root (carry out 3 iteration). **(07 Marks)**

OR

10 a. Find the rank of the matrix :

$$\begin{bmatrix} 221 & 22 & 23 & 24 \\ 22 & 23 & 24 & 25 \\ 23 & 24 & 25 & 26 \\ 24 & 25 & 26 & 27 \end{bmatrix}.$$

(06 Marks)

b. Solve the system of equations by Gauss – Jordan method,

$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3.$ **(07 Marks)**

c. Using Rayleigh power method find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}.$$

by taking $(1, 1, 1)^T$ as initial eigen vector (carryout 5 iterations). **(07 Marks)**

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