Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

CBCS SCHEME

USN | | | | | | | | | |

21MAT11

First Semester B.E. Degree Examination, Dec.2023/Jan.2024
Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. **(07 Marks)**

c. Find radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(07 Marks)**

OR

2 a. Show that for Cardioid $r = a(1 + \cos\theta)$, $\frac{\rho^2}{r}$ = constant. **(06 Marks)**

b. Find pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(07 Marks)**

c. Find radius of curvature of the curve $x = a\log(\sec t + \tan t)$, $y = a\sec t$. **(07 Marks)**

Module-2

3 a. Expand $\log(1 + \sin x)$ in powers of x upto terms containing x^4. **(06 Marks)**

b. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, then show that $\frac{\partial(uvw)}{\partial(xyz)} = 4$. **(07 Marks)**

c. If $u = e^{ax+by}.f(ax - by)$, show that $bu_x + au_y = 2abu$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. **(06 Marks)**

b. If $u = f(x - y, y - z, z - x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(07 Marks)**

Module-3

5 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(06 Marks)**

b. Find orthogonal trajectories of family of $r^n \cos n\theta = a^n$. **(07 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of body at the end of 3 minutes. **(07 Marks)**

OR

6 a. Solve $y(2xy + 1)dx - xdy = 0$. **(06 Marks)**

b. Prove that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

c. Find the general solution of equation $(px - y)(py + x) = a^2 p$ by reducing into Clairaut's form taking the substitution $X = x^2$ and $Y = y^2$. **(07 Marks)**

Module-4

7 a. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 2e^{3x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 + 2x$ **(07 Marks)**

c. Using method of variation of parameter, solve $y'' - 2y' + y = \frac{e^x}{x}$. **(07 Marks)**

OR

8 a. Solve $\frac{d^3y}{dx^3} + y = 65\cos(2x+1)$. **(06 Marks)**

b. Solve $\frac{d^4y}{dx^4} - 16y = e^x$. **(07 Marks)**

c. Solve $(1+x)^2 y'' + (1+x)y' + y = \sin[2\log(1+x)]$. **(07 Marks)**

Module-5

9 a. Find Rank of the matrix

$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

(06 Marks)

b. Find the values of λ and μ for which the system of equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has (i) unique solution (ii) infinitely many solution (iii) no solution. **(07 Marks)**

c. Solve the system of equations by Gauss elimination method $x+2y+z=3$, $2x+3y+2z=5$, $3x-5y+5z=2$. **(07 Marks)**

OR

10 a. Find Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(06 Marks)**

b. Apply Gauss Jordan method to solve $x+y+z=9$, $2x-3y+4z=13$, $3x+4y+5z=40$. **(07 Marks)**

c. Find the largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with initial approximate eigen vector $[1\ 1\ 1]^T$. **(07 Marks)**

* * * * *