CBCS SCHEME - Make-Up Exam

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BCS405A

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define Tautology and Contradiction. Show that for any proposition p and q the compound proposition $[p \rightarrow (p \vee q)]$ is a tautology and the compound proposition $[p \wedge (\sim p \wedge q)]$ is a contradiction by using truth table.	06	L2	CO1
	b.	Prove the following logical equivalence without using truth table: i) $[(p \rightarrow q) \wedge \{\sim q \wedge (r \vee \sim q)\}] \Leftrightarrow \sim(q \vee p)$ ii) $\{[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r)\} \Leftrightarrow r$	07	L2	CO1
	c.	Prove the following is a valid argument : $p \rightarrow (q \wedge r)$ $r \rightarrow s$ $\sim(q \wedge s)$ $\therefore \sim p$	07	L3	CO1
		OR			
Q.2	a.	For inverse of all real numbers, let p(x): $x \geq 0$, q(x) : $x^2 \geq 0$, r(x) : $x^2-3x-4=0$ s(x) : $x^2 - 3 > 0$. Determine the truth values of the following : (i) $\exists x, p(x) \wedge q(x)$ (ii) $\forall x, p(x) \rightarrow q(x)$ (iii) $\forall x, q(x) \rightarrow s(x)$ (iv) $\forall x, r(x) \vee s(x)$	06	L2	CO1
	b.	Test the validity of the following arguments: If a triangle has two equal sides, then it is isosceles If a triangle is isosceles, then it has two equal angles The triangle ABC does not have two equal angles ∴ ABC does not have two equal sides	07	L3	CO1
	c.	Give indirect proof for the following statement: (i) For all integers k and l, if kl is odd, then both k and l are odd. (ii) For all integers k and l, if k + l is even, then k and l are both even or both odd.	07	L3	CO1
		Module – 2			
Q.3	a.	Prove by Mathematical induction, for all integers $n \geq 1$ $1.2 + 2.3 + 3.4 + \ldots\ldots.+ n(n+1) = \frac{1}{3}n(n+1)(n+2)$	06	L2	CO2
	b.	A sequence $\{a_n\}$ is defined recursively by $a_0 = 1$ $a_1 = 1$ $a_2 = 1$ and $a_n = a_{n-1} + a_{n-3}$ for all $n \geq 3$. Prove that $a_{n+2} \geq (\sqrt{2})^n$ for all integer $n \geq 0$.	07	L2	CO2

Q.3	c.	Find the number of permutations of the letters of word MASSASAUGA. In how many of these, all four A's are together? How many of them begin with S?	07	L3	CO2
		OR			
Q.4	a.	From seven consonants and five vowels, how many sets consisting of four different consonants and three different vowels can be formed?	06	L2	CO2
	b.	Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$	07	L3	CO2
	c.	In how many ways can 10 identical pencils be distributed among 5 children in the following case: (i) There are no restrictions (ii) Each child gets atleast one pencil (iii) The youngest child gets atleast two pencils	07	L3	CO2
		Module – 3			
Q.5	a.	Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5 , 6} (i) Find how many functions are there from A to B. How many of these are one to one? (ii) Find how many functions are there from B to A. How many of these are onto? How many are one to one?	06	L2	CO3
	b.	State Pigeonhole Principle. How many persons must be chosen in order that at least five of them will have birthdays in the same calendar month?	07	L3	CO3
	c.	Prove that "A function $f : A \rightarrow B$ is invertible if and only if it is one to one and onto".	07	L2	CO3
		OR			
Q.6	a.	Let A = {1, 2, 3, 4} and R be the relation on A defined by xRy if and only if "x divides y", written as x/y. (i) Write down R as a set of ordered pairs. (ii) Draw the digraph of R, matrix M(R) (iii) Determine the indegree and outdegree of the vertices in the digraph.	06	L2	CO3
	b.	Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by (x_1, y_1) R (x_2, y_2) if and only if $x_1 + y_1 = x_2 + y_2$. (i) Verify that R is an equivalence relation on A×A (ii) Determine the equivalence classes of [(1, 3)] (iii) Determine the partition of A×A induced by R	07	L3	CO3
	c.	Let s = {1, 2, 3} and P(S) be the power set of S. On P(S) define the relation R by XRY if and only of $X \leq Y$. Show that this relation is a partial order on P(S). Draw its Hasse diagram.	07	L2	CO3

		Module – 4			
Q.7	**a.**	Define Derangement. Find the number of derangements of 1, 2, 3, 4. List them.	**06**	**L2**	**CO4**
	b.	Find the number of permutations of the letters a, b, c,.....x, y, z in which none of the patterns spin, game, path or net occurs.	**07**	**L3**	**CO4**
	c.	Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs.	**07**	**L3**	**CO4**
		OR			
Q.8	**a.**	Out of 30 students in a hostel, 15 study History, 8 study Economics and 6 study Geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects.	**06**	**L3**	**CO4**
	b.	Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$.	**07**	**L2**	**CO4**
	c.	Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.	**07**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	If $*$ is an operation on z defined by $x * y = x + y + 1$, prove that (z, $*$) is an abelian group.	**06**	**L2**	**CO5**
	b.	The symmetric group S_4 consists of all the permutations of the set A ={1, 2, 3, 4}. What is the order of S_4? What is the identity element in S_4? If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$ Verify the $(\alpha\,\beta)^{-1} = \beta^{-1}\,\alpha^{-1}$	**07**	**L3**	**CO5**
	c.	Let G be a group and let $J = \{x \in G \,/\, xy = yx \text{ for all } y \in G\}$ Prove that J is a subgroup of G.	**07**	**L2**	**CO5**
		OR			
Q.10	**a.**	Prove that the group $(z_4, +)$ is cyclic. Find all its generators.	**06**	**L3**	**CO5**
	b.	Prove the theorem "There exists a one to one correspondence between the elements of a subgroup and the elements of the left (right) coset thereof".	**07**	**L2**	**CO5**
	c.	State and prove Lagrange's theorem.	**07**	**L2**	**CO5**

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