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08EC046

M.Tech. Degree Examination, May/June 2010
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Solve the system of equations
$$y - 4z = 8, \quad 2x - 3y + 4z = 1, \quad 5x - 8y + 7z = 1$$ **(06 Marks)**

b. Find the LU – factorization of the matrix
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}.$$ **(07 Marks)**

c. Let v be a vector space over the field F. If S is any subset of v, then show that $S^\circ = [L(s)]^\circ$. **(07 Marks)**

2 a. Find the inverse of the matrix
$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$$ **(06 Marks)**

b. Diagonalise the matrix $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. **(07 Marks)**

c. Determine the scalar K such that $(KA)^T(KA) = 1$,

where $A = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Is the value unique? **(07 Marks)**

3 a. Let U and V be vector spaces over the field F. Let $\{v_1, v_2, \ldots\ldots v_n\}$ be a basis of V and let $u_1, u_2, \ldots\ldots u_n$ be any vectors. Then prove that there exists a unique linear mapping $T : V \to U$ such that $T(V_i) = u_i$. **(06 Marks)**

b. Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x,y,z) = (x + 2y - z,\ y+z,\ x+y-2z)$. Find a basis and dimension of i) the image of T ii) the kernel of T. **(07 Marks)**

c. Let T be a linear operator on R^3 defined by $T(x,y,z) = (2x, 4x - y, 2x + 3y - z)$
   i) Show that T is invertible
   ii) Find the formulae for T^{-1} and T^2. **(07 Marks)**

Important Note : 1. On completing your answ[...] compulsorily draw diagonal cross lines on the remaining [...]ages.
2. Any revealing of identific[...] appeal to evaluator and /or equations written eg, 42+8 = [...]ll be treated as malpractice.

D = David
S = Strang
G = Gilbert

4 a. Given a symmetric matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$. Diagonalise this. (10 Marks)

b. Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$. (10 Marks)

5 a. Let V(F) be a finite dimensional vector space and U_1 and U_2 be the two subspaces of V(F). If V(F) is the direct sum of U_1 and U_2 then prove that Dim V = dim U_1 + dim U_2. (10 Marks)

b. Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.

Find the characteristic polynomial and minimal polynomial of this. (10 Marks)

6 a. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$, subjected to the constraint $X^T X = 1$. (10 Marks)

b. Determine all the possible Jordan's canonical forms of a matrix of order 6 whose minimal polynomial is $m(\lambda) = (\lambda - 2)^2$. (10 Marks)

7 a. Find the singular value decomposition of matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. (10 Marks)

b. What are he generalized eigen vectors? Find the eigen vectors and the Jordan form of

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(10 Marks)

8 a. Determine the invariant subspaces of $A = \begin{bmatrix} 2 & -4 \\ 5 & -2 \end{bmatrix}$ viewed as linear operator on

i) IR^2 ii) C^2. (10 Marks)

b. By using the orthogonal projection determine the least square solution to this system of equations Ax = b where

$$A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}.$$

(10 Marks)

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