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USN

08EC046

M.Tech. Degree Examination, Dec.09/Jan.10

Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$. (06 Marks)

b. Using L-U decomposition method solve the system of equations
$6x_1 - 2x_2 - 4x_3 + 4x_4 = 2$
$3x_1 - 3x_2 - 6x_3 + x_4 = -4$
$-12x_1 + 8x_2 + 21x_3 - 8x_4 = 8$
$-6x_1 - 10x_3 + 7x_4 = 43$ (08 Marks)

c. Solve for the system of linear equations
$x_1 - 2x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$-4x_1 + 5x_2 + 9x_3 = -9$. (06 Marks)

2 a. Find the matrix 'P' which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Verify $P^{-1}AP = D$ where 'D' is a diagonal matrix, hence find A^6. (10 Marks)

b. Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. (10 Marks)

3 a. If V is an inner product space, then prove that for any vector α, β in V and any scalar C.
i) $\|c\alpha\| = |c|\,\|\alpha\|$
ii) $\|\alpha\| > 0$ for $\alpha \neq 0$
iii) $|(\alpha/\beta)| \leq \|\alpha\|\,\|\beta\|$
iv) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$. (06 Marks)

b. Prove that every finite dimensional inner product space has an orthonormal basis. (04 Marks)

c. If V is an inner product space and $\beta_1, \beta_2 \ldots \beta_n$ be any independent vector in V, then prove that it is possible to construct orthogonal vectors $\alpha_1, \alpha_2 \ldots \alpha_n$ in V such that for each $k = 1, 2, \ldots n$ the set $(\alpha_1 \ldots\ldots \alpha_k)$ is a basis for the subspace spanned by $\beta_1 \ldots\ldots \beta_k$. (10 Marks)

4 a. Construct a spectral decomposition of the matrix A that has orthogonal diagonalization.
$$A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}.$$ (06 Marks)

b. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into quadratic form with no cross product terms. (08 Marks)

c. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $X^T X = 1$. (06 Marks)

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8=50, will be treated as malpractice.

5 a. Let V be a n-dimensional vector space over the field F and W an m-dimensional vector space over F. Let B and B′ be ordered bases for V and W. For each linear transformation $T:V \to W$ show that there is a m x n matrix A such that $[T_\alpha]_{B'} = A[\alpha]_B$. **(06 Marks)**

b. Find the co-ordinates of (2, 3, 4, -1) relative to the ordered basis
$B = \{(1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1), (1, 0, 0, 0)\}$ for V_4. **(06 Marks)**

c. If U and W are two sub – spaces of a finite dimensional vector space V, then
$\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$. **(08 Marks)**

6 a. Define $T:V_3 \to V_2$ by the rule $T(x_1, x_2, x_3) = (x_1, -x_2, x_1 + x_3)$. Show that this is a linear map. **(06 Marks)**

b. Given a matrix $A = \begin{bmatrix} i & -i & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Determine the linear transformation $T:V_3(R) \to V_2(R)$ relative to the basis B_1 and B_2 given by

i) $B_1 = \{(1\ 1\ 1)\ (1\ 2\ 3)\ (1\ 0\ 0)\}$
$B_2 = \{(1\ 1)\ (1\ -1)\}$.

ii) B_1 and B_2 are standard basis of $V_3(R)$ and $V_2(R)$ respectively. **(10 Marks)**

c. Let T_1 and T_2 be linear operations on R^2 to R^2 defined as follows:
$T_1(x_1, x_2) = (x_2, x_1)$; $T_2(x_1, x_2) = (x_1, 0)$, show that T_1 and T_2 are not commutative. **(04 Marks)**

7 a. If T is a linear transformation from V into W where V and W are vector spaces over the field F, and V is finite dimensional, then prove that
rank (T) + nullity (T) = dim V. **(08 Marks)**

b. Let T be an invertible linear transformation on vector space V(F). Then show that $T^{-1}T = TT^{-1} = I$ **(06 Marks)**

c. Let 'f' be a linear functional on a vector space V(F), then prove the following:

i) f(0) = 0 where '0' on LHS is zero vector of V and '0' on RHS is zero element of F

ii) $f(-\alpha) = -f(\alpha)\ \forall\ \alpha \in V$. **(06 Marks)**

8 a. Find the least square solution of AX = B for $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$. **(04 Marks)**

b. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^\perp$ is the null space of E and $V = W \oplus W^\perp$. **(06 Marks)**

c. Let V be a n-dimensional vector space and let W be m-dimensional vector space over F. Show that the space $\perp(V, W)$ of linear transformation has the dimension mn. **(10 Marks)**

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