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08EC046

M.Tech. Degree Examination, Dec.08/Jan.09

Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions***

1 a. Solve the following system of equations:

$x+2y-3z=1$

$2x+5y-8z=4$

$3x+8y-13z=7$ by Gauss elimination method. (06 Marks)

b. Reduce the following matrix:

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$ to Row reduced Echelon form. (06 Marks)

c. Find the LU factorization with $l_{ii}=1$ for the matrix $A=\begin{bmatrix} 2 & 1 & 1 & -1 \\ 4 & 5 & 4 & -1 \\ -4 & 1 & 4 & 4 \\ 6 & -3 & 3 & 1 \end{bmatrix}$. (08 Marks)

2 a. Express M as a linear combination of the matrices A, B, C where

$M=\begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$, $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$ (06 Marks)

b. Prove that the set $W=\{(x,y,z)/x-3y+4z=0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$. (07 Marks)

c. Prove that the inverse of two subspaces of a vector space V is a subspace of V. Is it true in the case of union of two subspaces? Justify your answer. (07 Marks)

3 a. If α, β, γ are linearly independent in V(F), prove that the vectors α+β, α-β, α-2β+γ are also linearly independent. (06 Marks)

b. Prove that any two bases of a finite dimensional vector space V have the same number of elements. (06 Marks)

c. Let $T:U\rightarrow V$ be a linear map. Then prove that

i) R(T) is a subspace of V.

ii) N(T) is a subspace of U.

iii) T is 1 – 1 iff the null space (N(T)) is a zero subspace. (08 Marks)

4 a. Prove that $T:U\rightarrow V$ of a vector space U to a vector space V over the same field F is a linear transformation if and only if $\forall\ \alpha, \beta \in U$ and $C_1, C_2 \in F$

$T(C_1\alpha+C_2\beta)=C_1T(\alpha)+C_2T(\beta)$ (06 Marks)

b. Find the eigen space of the linear transformation

$T: R^3\rightarrow R^3$ defined by

T(x, y, z) = (2x+y, y-z, 2y+4z) (07 Marks)

c. Find the linear transformation relative to the bases,

$B_1=\{(1,1),(-1,1)\}$, $B_2=\{(1,1,1),(1,-1,1)(0,0,1)\}$ given the matrix $A_T=\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$. (07 Marks)

5 a. Verify Rank-nullity theorem for the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(x,y,z) = (x+y, x-y, 2x+z)$.
(06 Marks)

b. Let T be a linear transformation from a vector space U to a vector space V then T is non singular iff T is 1 – 1.
(07 Marks)

c. $T : R^2 \rightarrow R^2$ is given by $T(x,y) = (4x-2y, 2x+y)$. Verify whether T is non singular. Also find its inverse.
(07 Marks)

6 a. Find all invariant subspaces of $A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$ viewed as an operator on R^2.
(06 Marks)

b. Determine all possible Jordan canonical forms J for a linear operator $T : V \rightarrow V$ whose characteristic polynomial $\Delta T = (t-2)^5$ and whose minimal polynomial $m(t) = (t-2)^2$.
(07 Marks)

c. Find a least squares solution of the inconsistent system AX = b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.
(07 Marks)

7 a. Define an inner product space. Give one example. If V is an inner product space, then prove that for any vectors α, β in V $\|\alpha+\beta\| \leq \|\alpha\| + \|\beta\|$.
(06 Marks)

b. Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace of U of R^4 spanned by $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$, $v_3 = (1, -3, -4, -2)$.
(07 Marks)

c. Find aQR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
(07 Marks)

8 a. Find the maximum and minimum value of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $x^T x = 1$.
(06 Marks)

b. Make a change of variable $x = py$ that transforms the quadratic form $x_1^2 - 8x_1x_2 - 5x_2^2$ into a quadratic form with no cross product term.
(07 Marks)

c. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.
(07 Marks)

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