CBCS Scheme

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16SCN/SCE/SSE/LNI/SFC/SIT/SCS14

First Semester M.Tech. Degree Examination, Dec.2016/Jan.2017
Probability, Statistics and Queuing Theory

Time: 3 hrs. Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

1 a. State axioms of probability and Baye's theorem. **(04 Marks)**

b. A random variable X has the following probability function for various values of X.

$X = x_i$	0	1	2	3	4	5	6	7
$P(x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find: i) the value of k; ii) E(x) and iii) $P(x < 6)$. **(06 Marks)**

c. The joint density function of two continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} kxy, & 0 \le x \le 4, \quad 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find: i) the value of k; ii) E(XY). **(06 Marks)**

OR

2 a. A cellphone is rejected if the design is faulty or not. The probability that the design is faulty is 0.1 and that the cellphone is rejected because of faulty design is 0.95 and otherwise is 0.45. If a cellphone is rejected, what is the probability that it is due to faulty design? **(04 Marks)**

b. Suppose that the error in the reaction temperature in °C, for a controlled laboratory experiment is a random variable X having the p.d.f.

$$f(x) = \begin{cases} x^2/3, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find: i) F(x) and ii) Use it to evaluate $P(0 < x \le 1)$. **(06 Marks)**

c. The joint probability distribution of two random variables X and Y is given below:

X \ Y	3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find:
i) Marginal distribution of X and Y
ii) E(X) and E(Y)
iii) Covariance of X and Y. **(06 Marks)**

Module-2

3 a. If X is an exponential variate with mean 5. Find: i) $P(x \le 0 \text{ or } x \ge 1)$; ii) $P(0 < x < 1)$. **(04 Marks)**

b. Derive mean and variance of the binomial distribution. **(06 Marks)**

c. If X has a geometric distribution with parameter P, find i) P(x is even) ii) P(x is odd). **(06 Marks)**

OR

4 a. A company receives 3 complaints per day on an average. What is the probability of receiving more than one complaint on a particular day? **(04 Marks)**

b. Derive mean and variance of the uniform distribution. **(06 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean = 70 and standard deviation = 5. Find the number of students whose marks will be,
i) Less than 65 ii) Between 65 and 75.
(Given: $P(0 < z < 1) = 0.3413$). **(06 Marks)**

Module-3

5 a. Consider a random process x(t) defined by $x(t) = A\cos(wt + \theta)$ where A and θ are independent and uniform random variables over (-k, k) and $(-\pi, \pi)$, respectively. Find the mean of x(t). **(04 Marks)**

b. Let X(t) and Y(t) be jointly wide-sense stationary processes with constant means μ_x and μ_y respectively and cross-correlation function $R_{xy}(\tau)$. Then prove that,
i) $R_{xy}(-\tau) = R_{yx}(\tau)$.
ii) $|R_{xy}(\tau)| \le \sqrt{R_{xx}(0)\, R_{yy}(0)}$. **(06 Marks)**

c. Define Markov process. Find the unique fixed probability vector of the following stochastic matrix, $A = \begin{bmatrix} 1/3 & 2/3 \\ 1 & 0 \end{bmatrix}$. **(06 Marks)**

OR

6 a. Define random process and give its classification. **(04 Marks)**

b. A stationary random process X = {x(t)} with mean 3 has auto-correlation function $R(\tau) = 16 + 9e^{-|\tau|}$. Find the standard deviation of the process. **(06 Marks)**

c. Let $\{X(t) : t \ge 0\}$ be a Poisson process with parameter λ. Suppose that each arrival is registered with probability P independent of other arrivals. Let $\{Y(t) : t \ge 0\}$ be the process of registered arrivals. Prove that Y(t) is a Poisson process with parameter λ_p. **(06 Marks)**

Module-4

7 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance [Given $Z_{0.05} = 1.96$]. **(04 Marks)**

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 – test.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
Number of accidents	14	16	08	12	11	09	14	84

($\chi^2_{0.05} = 12.6$) **(06 Marks)**

c. Two types of batteries are tested for their length of life and the following results were obtained. Battery A: $n_1 = 10$, $\overline{x_1} = 500\text{hrs}$, $s_1^2 = 100$ and Battery B: $n_2 = 10$, $\overline{x_2} = 560\text{hrs}$, $s_2^2 = 121$. Compute student's t-test and test whether there is a significant difference in the two means ($t_{0.05} = 2.101$). **(06 Marks)**

OR

8 a. A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch standard deviation 0.3. Can it be said that the machine is producing nails as per specification. ($t_{0.05} = 2.064$). **(04 Marks)**

b. One type of aircraft is found to develop engine trouble in 5 flights out of a total of 100 and another type in 7 flight out of a total of 200 flights. Is there a significant difference in the two types of aircrafts so far as engine defects are concerned? **(06 Marks)**

c. Define F-distribution. Two random samples gave the following:

$n_1 = 10$; $\Sigma(x_i - \bar{x})^2 = 90$; $n_2 = 12$; $\Sigma(y_i - \bar{y})^2 = 108$. Test whether the samples came from the same population. ($F_{0.05} = 3.11$ for the degrees of freedom (9, 11)). **(06 Marks)**

Module-5

9 a. A beauty salon has 2 barbers and 6 chairs to accommodate waiting customers. Potential customers, who arrive when all the 6 chairs are full, leave the salon immediately. Customers arrive at the salon at the average rate of 10 per hour and spend an average of 10 minutes in the barber's chair. The arrival process is Poisson and the service time is an exponential random variable. Find the probability of no customer in the beauty salon. **(04 Marks)**

b. State the assumptions of M/M/1 : (∞/FIFO) queuing model and find the probability that there are at least n customers in the system. **(06 Marks)**

c. A telephone exchange has two long-distance operators. It has been found that long telephone calls arrive according to a Poisson distribution at an average rate of 15 per hour. The length of service of these calls has been shown to be exponentially distributed with mean length of 2 minutes. Find the:

i) Probability that a customer will have to wait for his long distance call.

ii) Expected number of customers in the system. **(06 Marks)**

OR

10 a. Patients arrive at a doctor's clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than nine patients. Examination time per patient is exponential with mean rate of 20 per hour. Find out:

i) Probability that an arriving patient will not wait.

ii) Effective arrival rate. **(04 Marks)**

b. Derive the Little's formulae for the M/M/1 : ∞/FIFO model, that is the relations between L_s, L_q, W_s and W_q as follows:

i) $L_s = \lambda W_s$, ii) $L_q = \lambda W_q$, iii) $W_s = W_q + 1/\mu$, iv) $L_s = L_q + \lambda/\mu$. **(06 Marks)**

c. A road transport company has 2 reservation clerks serving the customers. The customers arrive in a Poisson fashion at the rate of 8 per hour. The service time for each customer is exponentially distributed with mean 10 minutes. Find the probability that a customer has to wait for service. **(06 Marks)**

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