

**Third Semester B.E./B.Tech. Degree Examination, June/July 2024**

## AV Mathematics – III for EC/BM Engineering

Time: 3 hrs.

Max. Marks: 100

**Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.**

**2. VTU Formula Hand Book and statistical table are permitted.**

3. *M* : Marks , *L*: Bloom's level , *C*: Course outcomes.

Module – 1				M	L	C															
Q.1	a.	Find the Fourier series for $f(x) = \begin{cases} -K, & \text{in } (-\pi, 0) \\ K & \text{in } (0, \pi) \end{cases}$ and hence deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$	6	L2	CO1																
	b.	Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 < x < 1$ .	7	L2	CO1																
	c.	Express y as a Fourier series upto the first harmonics given the following values: <table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>4</td><td>8</td><td>15</td><td>7</td><td>6</td><td>2</td></tr></table>	x	0	1	2	3	4	5	y	4	8	15	7	6	2	7	L3	CO1		
x	0	1	2	3	4	5															
y	4	8	15	7	6	2															
OR																					
Q.2	a.	Find the Fourier series for $f(x) = x - x^2$ in $-1 < x < 1$ .	6	L2	CO1																
	b.	Show that half range sine series of $f(x) = \pi x - x^2$ in the interval $(0, \pi)$ is $\frac{8}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin(2n+1)x$	7	L2	CO1																
	c.	Obtain the Fourier series of y upto 2 <sup>nd</sup> harmonics f(x) is given by <table border="1"><tr><td>x</td><td>0</td><td><math>\pi/3</math></td><td><math>2\pi/3</math></td><td><math>\pi</math></td><td><math>4\pi/3</math></td><td><math>5\pi/3</math></td><td><math>2\pi</math></td></tr><tr><td>f(x)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table>	x	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$	f(x)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98	7	L3	CO1
x	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$														
f(x)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98														
Module – 2																					
Q.3	a.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, &  x  < 1 \\ 0 &  x  \geq 1 \end{cases}$ and hence find the value of $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$	6	L2	CO2																
	b.	Find the Fourier sine and cosine transform of $f(x) = e^{-\alpha x}$ , $\alpha > 0$ .	7	L2	CO2																
	c.	Solve the integral equation $\int_0^{\infty} f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1 - \alpha, & 0 \leq \alpha \leq 1 \\ 0, & \alpha > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$ .	7	L3	CO2																

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OR

Q.4	a.	Find the Fourier transform of $e^{-a^2x^2}$ , $a > 0$ .	6	L2	CO2
	b.	Find the Fourier sine transform of $f(x) = e^{- x }$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$ , $m > 0$	7	L2	CO2
	c.	Find the discrete Fourier transform of the sequence $\{1, 2, 1, 3\}^T$ .	7	L3	CO2

Module – 3

Q.5	a.	Obtain the Z-transform i) $\cos n\theta$ ii) $\sin n\theta$ .	6	L2	CO3
	b.	Find the inverse Z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$	7	L2	CO3
	c.	Solve by using Z-transforms : $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$ .	7	L3	CO3

OR

Q.6	a.	Find the Z-transform of $2n + \sin\left(\frac{n\pi}{4}\right) + 1$	6	L2	CO3
	b.	Find the inverse Z-transform of $\frac{4z^2 - 2z}{(z-1)(z-2)^2}$ .	7	L2	CO3
	c.	If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(Z-1)^4}$ find the value of $u_0, u_1, u_2$ .	7	L3	CO3

Module – 4

Q.7	a.	Solve $(D^4 + 8D^2 + 16)y = 0$ .	6	L1	CO4
	b.	Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = e^{3t} \cosh 2t$ .	7	L2	CO4
	c.	Solve $x^3 + x^2y'' + xy' + 8y = 65 \cos(\log x)$ .	7	L3	CO4

OR

Q.8	a.	Solve $y'' + 9y = \cos 2x \cos x$ .	6	L2	CO4
	b.	Solve $(2x+1)^2 y'' - 2(2x+1)y' - 12y = 6x + 5$ .	7	L2	CO4
	c.	In an LCR circuit, the charge $q$ on a plate of a condenser is given by $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = E \sin pt$ . Solve the equation for $q$ .	7	L3	CO4



## Module – 5

Q.9	a.	Fit a straight line for the following data: <table><tr><td>x</td><td>50</td><td>70</td><td>100</td><td>120</td></tr><tr><td>y</td><td>12</td><td>15</td><td>21</td><td>25</td></tr></table>	x	50	70	100	120	y	12	15	21	25	6	L1	CO5												
x	50	70	100	120																							
y	12	15	21	25																							
	b.	Obtain the lines of regression and hence find the coefficient of correlation for the data: <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td></tr></table>	x	1	2	3	4	5	6	7	y	9	8	10	12	11	13	14	7	L2	CO5						
x	1	2	3	4	5	6	7																				
y	9	8	10	12	11	13	14																				
	c.	Compute the rank correlation coefficient for the following data: <table><tr><td>x</td><td>68</td><td>63</td><td>75</td><td>50</td><td>62</td><td>80</td><td>78</td><td>40</td><td>55</td><td>60</td></tr><tr><td>y</td><td>62</td><td>58</td><td>68</td><td>45</td><td>81</td><td>60</td><td>68</td><td>48</td><td>50</td><td>70</td></tr></table>	x	68	63	75	50	62	80	78	40	55	60	y	62	58	68	45	81	60	68	48	50	70	7	L3	CO5
x	68	63	75	50	62	80	78	40	55	60																	
y	62	58	68	45	81	60	68	48	50	70																	
OR																											
Q.10	a.	An experiment on life time 't' of cutting tool at different cutting speeds v(units) are given below <table><tr><td>Speed (v)</td><td>350</td><td>400</td><td>500</td><td>600</td></tr><tr><td>Life (t)</td><td>61</td><td>26</td><td>7</td><td>2.6</td></tr></table> Fit a relation of the form $v = at^b$ .	Speed (v)	350	400	500	600	Life (t)	61	26	7	2.6	6	L2	CO5												
Speed (v)	350	400	500	600																							
Life (t)	61	26	7	2.6																							
	b.	The following data gives the age of husband (x) and the age of wife (y) in years. Form the 2 regression lines and calculate the age of husband corresponding to 16 years of age of wife. <table><tr><td>x</td><td>36</td><td>23</td><td>27</td><td>28</td><td>28</td><td>29</td><td>30</td><td>31</td><td>33</td><td>35</td></tr><tr><td>y</td><td>29</td><td>18</td><td>20</td><td>22</td><td>27</td><td>21</td><td>29</td><td>27</td><td>29</td><td>28</td></tr></table>	x	36	23	27	28	28	29	30	31	33	35	y	29	18	20	22	27	21	29	27	29	28	7	L2	CO5
x	36	23	27	28	28	29	30	31	33	35																	
y	29	18	20	22	27	21	29	27	29	28																	
	c.	If the coefficient of correlation between the variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}(3/5)$ . Show that $\sigma_y = 2\sigma_x$ .	7	L3	CO5																						

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