**CBCS SCHEME**

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**BCS405A**

# Fourth Semester B.E./B.Tech. Degree Examination, June/July 2024

# Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Define tautology. Prove that for any propositions p, q, r the compound proposition. $[(p \wedge \neg q) \rightarrow r] \rightarrow [p \rightarrow (q \vee r)]$ is a tautology | 06 | L2 | CO1 |
| | b. | Test whether the following is a valid argument:<br>If Ram studies then he will pass $12^{th}$.<br>If Ram passes $12^{th}$ then his father gifts him a bike.<br>If Ram doesn't play video game then he will pass $12^{th}$.<br>Ram did not get a bike.<br>$\therefore$ Ram played video game. | 07 | L3 | CO1 |
| | c. | Give direct proofs of the statements:<br>i) If k and $l$ are odd then $k + l$ is even.<br>ii) If k and $l$ are odd then $kl$ is odd. | 07 | L2 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | Define (i) Proposition (ii) Open statement (iii) Quantifiers | 06 | L2 | CO1 |
| | b. | Using the laws of logic, prove the following logical equivalence:<br>$[(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p] \Leftrightarrow p \wedge \neg q$ | 07 | L2 | CO1 |
| | c. | Write the following statement in symbolic form and find its negation:<br>"If all triangles are right angled then no triangle is equilateral". | 07 | L2 | CO1 |
| | | **Module – 2** | | | |
| Q.3 | a. | Prove by using mathematical induction.<br>$1^2 + 2^2 + \ldots\ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ | 06 | L2 | CO1 |
| | b. | How many words can be made with or without meaning from the letters of the word "STATISTICS"? In how many of these a and c are adjacent? In how many vowels are together? | 07 | L3 | CO2 |
| | c. | Find the coefficient of $x^3y^8$ in the expansion of $(2x - y)^{11}$. | 07 | L2 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | Obtain the recursive definition for the sequence in each of the following cases:<br>(i) $a_n = 5n$ (ii) $a_n = 3n + 7$ (iii) $a_n = n^2$ (iv) $a_n = 2 - (-1)^n$ | 06 | L2 | CO2 |
| | b. | A woman has 11 close relations and wishes to invite 5 of them to dinner. In how many ways can she invite them if (i) there is no restriction on her choice. (ii) 2 persons will not attend separately (iii) 2 persons will not attend together. | 07 | L3 | CO2 |
| | c. | In how many ways can we distribute 7 apples and 5 oranges among 3 children such that each child gets atleast one apple and one orange? | 07 | L3 | CO2 |

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| | | **Module – 3** | | | |
| **Q.5** | **a.** | State pigeon hole principle. Using pigeon hole principle find the minimum number of persons chosen so that atleast 5 of them will have their birthday in the same month. | 06 | L3 | CO3 |
| | **b.** | Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5}. Find the number of 1-1 functions and onto functions from (i) A to B (ii) B to A | 07 | L2 | CO3 |
| | **c.** | Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$.<br>(i) Verify that R is an equivalence relation<br>(ii) Determine the equivalence class of [(2, 4)] | 07 | L2 | CO3 |
| | | **OR** | | | |
| **Q.6** | **a.** | Consider the functions f and g from R to R defined by f(x) = 2x + 5 and g(x) = ½ (x – 5). Prove that g is inverse of f. | 06 | L2 | CO3 |
| | **b.** | Let A = {1, 2, 3, 4} and R be the relation on A defined by xRy if and only if x < y. Write down R as a set of ordered pairs. Write the relation matrix and draw the digraph. List out the in degrees and out degrees of every vertex. | 07 | L2 | CO3 |
| | **c.** | Let A = {1, 2, 3, 6, 9, 12, 18} and define R on A by xRy iff 'x divides y'. Prove that (A, R) is a POSET. Draw the Hasse diagram for (A, R). | 07 | L2 | CO3 |
| | | **Module – 4** | | | |
| **Q.7** | **a.** | How many integers between 1 and 300 (inclusive) are divisible by (i) atleast one of 5, 6 or 8. (ii) None of 5, 6 and 8. | 06 | L3 | CO4 |
| | **b.** | At a restaurant 10 men handover their umbrellas to the receptionist, In how many ways can their umbrellas be returned so that (i) no man receives his own umbrella. (ii) atleast one gets his own umbrella. (iii) atleast two gets their own umbrellas. | 07 | L3 | CO4 |
| | **c.** | The number of virus affected files in a system is 1000 (to start with) and this increases by 250% every 2 hours. Use a recurrence relation to determine the number of virus affected files in the system after 12 hours. | 07 | L3 | CO4 |
| | | **OR** | | | |
| **Q.8** | **a.** | In how many ways one can arrange the letters of the word "CORRESPONDENTS" so that there are (i) no pair (ii) atleast 2 pairs of consecutive identical letters. | 06 | L3 | CO4 |
| | **b.** | 4 persons $P_1$, $P_2$, $P_3$, $P_4$ who arrive late for a dinner party find that only one chair at each of five tables $T_1$, $T_2$, $T_3$, $T_4$ and $T_5$ is vacant. $P_1$ will not sit at $T_1$ or $T_2$. $P_2$ will not sit at $T_2$. $P_3$ will not sit at $T_3$ or $T_4$. $P_4$ will not sit at $T_4$ or $T_5$. Find the number of ways they can occupy the vacant chairs. | 07 | L3 | CO4 |
| | **c.** | Solve the recurrence relation<br>$a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$ with $a_0 = 5$, $a_1 = 12$. | 07 | L2 | CO4 |
| | | **Module – 5** | | | |
| **Q.9** | **a.** | If ∗ is an operation on Z defined by xy = x + y + 1, prove that (Z, ∗) is an abelian group. | 06 | L2 | CO5 |
| | **b.** | Explain Klein-4 group with example. | 07 | L2 | CO5 |
| | **c.** | State and prove Lagrange's theorem. | 07 | L2 | CO5 |
| | | **OR** | | | |
| **Q.10** | **a.** | Prove that intersection of two subgroups of a group G is also a subgroup of G. | 06 | L2 | CO5 |
| | **b.** | Prove that $(Z_4, +)$ is a cyclic group. Find all its generators. | 07 | L2 | CO5 |
| | **c.** | Let G = $S_4$ for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$<br>Find the subgroup H = <α> determine the left cosets of H in G. | 07 | L3 | CO5 |

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