CBCS SCHEME

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22MATEE11/22MAT11/22EPE11

First Semester M.Tech. Degree Examination, Dec.2023/Jan.2024
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Find the inverse of the following matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$, using the partition method. Also, find the solution of the system of equations, $x_1 + x_2 + x_3 = 1$; $4x_1 + 3x_2 - x_3 = 6$; $3x_1 + 5x_2 + 3x_3 = 4$	10	L2	CO1
	b.	Use the Given's method to reduce the following matrix to tridiagonal form $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$	10	L3	CO1
		OR			
Q.2	a.	Solve the following equations by using Relaxation method $9x - 2y + z = 50$; $x + 5y - 3z = 18$; $-2x + 2y + 7z = 19$	10	L2	CO1
	b.	Find all the eigenvalues and eigenvectors of the matrix, $A = \begin{bmatrix} 1 & 1 & 0.5 \\ 1 & 1 & 0.25 \\ 0.5 & 0.25 & 2 \end{bmatrix}$ by using Given's method.	10	L3	CO1
		Module – 2			
Q.3	a.	Define: (i) Vector space (ii) Subspace. Show H is a subspace of V where $V = \{V_1, V_2\}$ and $H = \text{Span}\,\{V_1, V_2\}$.	07	L1	CO2
	b.	If W_1 and W_2 are subspaces of the vector space V(F) then show that: (i) $W_1 + W_2$ is a subspace of V(F) (ii) $W_1 + W_2 = \{W_1 \cup W_2\}$.	07	L1	CO2
	c.	Show that the three vectors (1, 1, –1), (2, –3, 5) and (–2, 1, 4) of $\mathbb{R}^3$ are linearly independent.	06	L2	CO2
		OR			

			Marks	Level	CO
Q.4	a.	Define a linear transformation. A transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x) = Ax$, so that $T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$. Find: (i) T(u), the image of u under the transformation T. (ii) Find an x in $\mathbb{R}^2$ whose image under T is b. (iii) Determine whether c is in the range of transformation T.	07	L3	CO2
	b.	Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Prove that T is one-to-one if and only if the equation $T(x) = 0$ has only the trivial solution.	07	L1	CO2
	c.	Show that the vectors (1, 2, 1), (2, 1, 0), (1, –1, 2) form a basis of $\mathbb{R}^3$.	06	L2	CO2
		Module – 3			
Q.5	a.	Find an orthonormal basis of a subspace R^3 spanned by the vectors $S = \{u_1 = (1, 1, 1), u_2 = (-1, 0, -1), u_3 = (-1, 2, 3)\}$ by applying Gram-Schmidt orthogonalization process.	10	L3	CO3
	b.	Find the QR decomposition of the matrix, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ by applying Gram-Schmidt method.	10	L2	CO3
		OR			
Q.6	a.	Determine a least squares solution to $Ax = b$, where $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & 1 & 2 \\ -2 & 3 & 4 & 1 \\ 4 & 2 & 1 & 0 \\ 0 & 2 & 1 & 3 \\ 1 & -1 & 2 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \\ 3 \\ 5 \end{bmatrix}$	10	L2	CO3
	b.	Find the singular value decomposition of the given matrix, $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$.	10	L3	CO3
		Module – 4			
Q.7	a.	A balanced coin is tossed nine times. Find the probabilities of each of the following events: (i) Exactly 3 heads occurred (ii) Atleast 3 heads occurred (iii) Atleast 3 heads and atleast 2 tails occurred.	07	L3	CO4

b. A Gaussian random variable has a probability density function of the form $f_x(x) = c\exp[(-2x^2 + 3x + 1)]$. (07, L3, CO4)

(i) Find the value of the constant c.

(ii) Find the values of the parameter m and σ for this Gaussian random variable.

c. The current (in mA) measured in a piece of copper wire is known to follow a uniform distribution over the interval [0, 25]. Write down the formula for the probability density function f(x) of the random variable X representing the current. Calculate the mean and variance of the distribution. (06, L2, CO4)

OR

Q.8 a. In a normal distribution 15% items are below 35 and 10% items are above 65. Find the mean and standard deviation, $\phi(1.04) = 0.35$ and $\phi(1.28) = 0.4$. (07, L2, CO4)

b. The joint probability distribution of two random variables X and Y is as follows: (07, L3, CO4)

X \ Y	–4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute the (i) COV (X, Y) (ii) ρ(X, Y)

c. The probability that a news reader commits no mistake in reading news is $1/e^3$. Find the probability that on a particular news broadcast. The reader commits: (06, L2, CO4)

(i) only 2 mistakes (ii) more than 3 mistakes (iii) atmost 3 mistakes

Module – 5

Q.9 a. Define: (07, L1, CO5)

(i) Characteristic function

(ii) Moment generating function

(iii) Probability generating function

Find the probability generating function of f(x) given by $f(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$

b. Suppose a source sends symbols from a three-letter alphabet with $X \in \{a, b, c\}$ and $P_a = \frac{1}{2}$, $P_b = \frac{1}{4}$, $P_c = \frac{1}{4}$ are the source symbol probabilities. (07, L2, CO5)

(i) Determine the entropy of this source.

(ii) Give a source code that has an average code word length that matches the entropy.

c. Calculate the mean and second moment for a random variable with a uniform probability density function given as (06, L3, CO5)

$$f_X(x) = \begin{cases} 1/a, & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

OR

Q.10	a.	Let c_n be the n^{th} central moment of a random variable and μ_n be its n^{th} moment. Find a relationship between c_n and μ_k, k = 0, 1, 2, …… n.	07	L1	CO5
	b.	Calculate the first moment and second moment for a discrete random variable that has a binomial distribution with probability mass function given by, $$P_x(k) = \binom{n}{k} p^k (1-p)^{n-k},\ k = 0, 1, 2, \ldots, n$$	07	L2	CO5
	c.	If 'x' be a random variable with probability generating function $P_x(t)$, find the probability generating function of (i) x + 2 (ii) 2x.	06	L2	CO5

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