CBCS SCHEME

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22SCN/SAM/SCS/SDS/SAD/VSA/VSC/VCS/SCR/ISS11

First Semester M.Tech Degree Examination, Dec.2023/Jan.2024
Mathematical Foundation of Computer Science

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define vector space and given an example.	08	L1	CO1
	b.	Prove that : i) $\alpha 0 = 0$, for all α in F ii) $0v = 0$ for all v in v iii) $\alpha(u - v) = \alpha u - \alpha v$, for au $\alpha \in$ F and for all u, v in F iv) $\alpha v = 0 \Rightarrow \alpha = 0$ or $v = 0$, $\alpha \in F$, $v \in v$.	12	L2	CO1
		OR			
Q.2	a.	If ω_1 and ω_2 are two subspaces of a vector space V(F), prove that their sum ω_1 and ω_2 is a subspace of V(F).	7	L2	CO1
	b.	Define the following terms, i) Linearly independent ii) Basis and dimension iii) Linear transformation.	7	L2	CO1
	c.	If u, v, w be vector spaces over the field F and $T_1 : v \to w$, $T_2 : u \to v$ be two linear transformations prove that T_1T_2 is a linear transformation u to w.	6	L2	CO1
		Module – 2			
Q.3	a.	Define orthogonality with example and prove that x is orthogonal to y and y orthogonal to z. If for the vector $x = (1, 2, 3)^T$, $y = (-3, -6, 5)^T$, $z = (0, 5, 6)^T$ in $\mathbb{R}^3$.	10	L2	CO2
	b.	Use the Gram-Schmidt orthogonal process to construct an orthogonormalset of vectors from the linearly independent set $\{x_1, x_2, x_3\}$, where $x_1 = (1, 1, 0)^T$, $x_2 = (0, 1, 1)$, $x_3 = (1, 0, 1)^T$.	10	L2	CO2
		OR			
Q.4	a.	Find the least – squares solution of the inconsistent system Ax = b for : $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.	10	L2	CO2
	b.	Find the St-line of best fit to the data : <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>4</td><td>6</td><td>3</td><td>5</td><td>7</td></tr></table>	10	L2	CO2
		Module – 3			
Q.5	a.	Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.	10	L2	CO3
	b.	Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.	10	L2	CO3

		OR			
Q.6	**a.**	Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -1 \end{bmatrix}$.	10	L2	CO3
	b.	Compute the same mean and the co-variance of matrix $\begin{bmatrix} 1 & 4 & 7 & 8 \\ 2 & 2 & 8 & 4 \\ 1 & 13 & 1 & 5 \end{bmatrix}$.	10	L2	CO3
		Module – 4			
Q.7	**a.**	Define the testing hypothesis and the nine items of a sample gave the values 45, 47, 50, 52, 45, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5?	10	L2	CO4
	b.	Define goodness of fit for λ^2. Distribution and the theory predicts the proportions of beans in the four groups G, G_2, G_3, G_4 should be in the ratio 9 : 3 : 3 : 1. In expt with 1600 beans the number in the groups were 882, 313, 287 and 118. Does the experimental result support the theory?	10	L2	CO4
		OR			
Q.8	**a.**	Explain the analysis of variance of one way classification.	10	L2	CO4
	b.	Two independent samples of sizes 9 and 8 gave the sum of squares of deviations from their respective means as 160 and 91 respectively. Can the samples be regarded as drawn from the two normal population with same variance? (Given $F_{0.05}(8, 7) = 3.73$, $F_{0.05}(7, 8) = 3.50$).	10	L2	CO4
		Module – 5			
Q.9	**a.**	Define Fourier series and find the Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$	10	L2	CO5
	b.	Prove that $\int_{-\ell}^{\ell} \lvert f(x)\rvert^2 dx = \ell\left\{\frac{1}{2}a_0^2 + \sum_{x=1}^{\infty}(a_x^2 + b_x^2)\right\}$ using Parseval's formula.	10	L2	CO5
		OR			
Q.10	**a.**	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \lvert x\rvert \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$ Hence evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx$.	10	L3	CO5s
	b.	Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. Hence derive Fourier sine transform of $d(x) = \frac{x}{1+x^2}$.	10	L2	CO5

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