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18EC45

Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025
Signals and Systems

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Check whether the following signals are periodic or not. If periodic find the fundamental period.

i) $x(t) = \sin\left(\dfrac{\pi}{3}t\right) + 2\cos\left(\dfrac{8\pi}{3}t\right)$

ii) $x[n] = \sin\left(\dfrac{3\pi n}{4}\right)\sin\left(\dfrac{\pi}{3}n\right)$

iii) $x(t) = e^{j\frac{12\pi}{7}t} + e^{j\frac{12\pi}{5}t}$ **(10 Marks)**

b. Calculate the energy and power of the following signals as applicable:
i) $x[n] = (j)^n + (j)^{-n}$
ii) $x[n] = 8\,(0.5)^n\,U[n]$ **(06 Marks)**

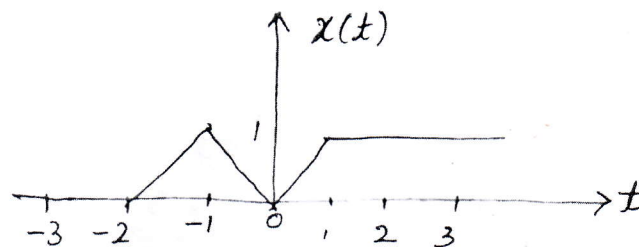c. Determine and sketch the even and odd parts of the signal depicted in Fig.Q.1(c).   **(04 Marks)**

Fig.Q.1(c)

OR

2 a. A trapezoidal pulse x(t) is defined by

$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t+5, & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$

is applied to a differentiator having the input-output relation $y(t) = \dfrac{dx(t)}{dt}$. Find the energy of y(t).   **(04 Marks)**

b. Show that the product of two even signals or two odd signals is an even signal, while the product of an even and odd signal is an odd signal.   **(06 Marks)**

c. From the signals indicated in Fig.Q.2(c), derive the following signals:
   i) $x(t-1) \, y(-t)$
   ii) $x(t) \, [\delta(t-1) + \delta(t-2)] + y(t) \, s(t+2)$
   iii) $x(t) \, y(t-1)$
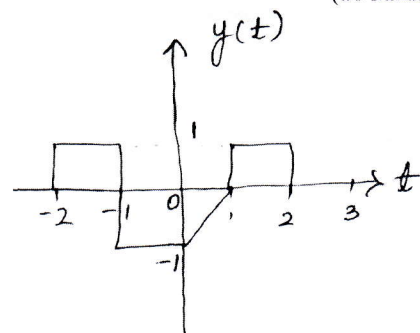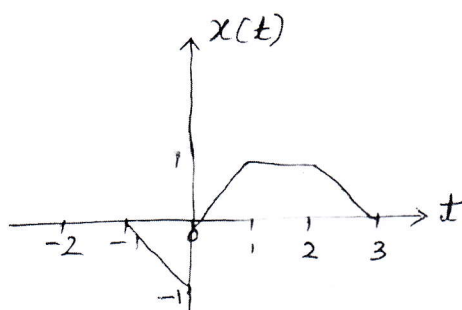   iv) $x(t) \, y(-t-1)$ **(10 Marks)**

Fig.Q.2(c)

Module-2

3 a. Determine whether the system described by the following input-output relation is
      i) Linear ii) Causal iii) Time-invariant iv) Memoryless v) Stable.
      $$y(t) = 2x(t) + 3$$ **(06 Marks)**

   b. Perform the convolution operation on the following signals:
      $$x(t) = e^{-2t} \, u(t)$$
      $$h(t) = \mu(t+2)$$ **(08 Marks)**

   c. Show that
      i) $\left[x(n) * h_1(n)\right] * h_2(n) = x(n) * \left[h_1(n) * h_2(n)\right]$
      ii) $x(n) * h(n) = h(n) * x(n)$ **(06 Marks)**

OR

4 a. Given $x[n] = \alpha^n \, U[n]$ and $h[n] = \beta^n \, U[n]$, perform $x[n] * h[n]$.   **(08 Marks)**

   b. Determine whether the following systems represented by input-output relations are
      invertible. If invertible then represent their inverse system

      i) $y(t) = s^{to} \{x(t)\}$ ii) $y(t) = \dfrac{1}{L} \displaystyle\int_{-\infty}^{t} x(z) \, dz$

      iii) $y(t) = x^2(t)$ iv) $y(t) = 2x(t)$ **(08 Marks)**

   c. Perform convolution operations on the following signals and sketch the output:
      $$x[n] = \delta(n+1) + 2\delta(n) + 3\delta(n-1) - 2\delta(n-2) + \delta(n-3)$$

      $$h[n] = \left(\frac{1}{2}\right)^n \left[U[n] - U[n-4]\right]$$ **(04 Marks)**

Module-3

5 a. Determine whether the following systems represented by impulse responses are stable,
      causal and memoryless.
      i) $h(n) = U(n-1) - U(n-5)$
      ii) $h(t) = e^{-t} \, \mu(+t)$
      iii) $h(n) = 0.5^{|n|}$
      iv) $h(t) = \mu(t-1)$ **(08 Marks)**

b. Show that step response of an LTI system is running integral of impulse response. **(04 Marks)**

c. Evaluate the Fourier series representation for the signal $x(t) = \sin(2\pi t) + \cos(3\pi t)$. Sketch the magnitude and phase spectra. **(08 Marks)**

OR

6   a. State and prove convolution property for continuous time Fourier series. **(04 Marks)**

b. Find the Fourier series representation for the given signal and draw its magnitude and phase spectra. **(10 Marks)**
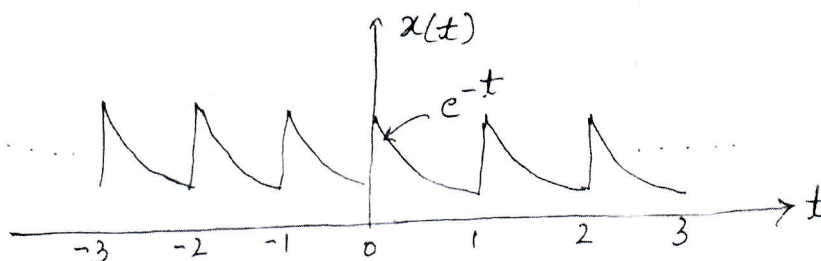

Fig.Q.6(b)

c. Find the unit step response of the systems given by their impulse responses

   i) $h(n) = \left(\dfrac{1}{2}\right)^n U(n)$ ii) $h(t) = e^{-|t|}$ **(06 Marks)**

Module-4

7   a. State and prove the following properties applicable to continuous time Fourier transform
   i) Time shift   ii) Frequency differentiation **(06 Marks)**

b. Find the continuous time Fourier transform of $x(t) = e^{-at} u(t)$; $a > 0$. Draw its magnitude and phase spectra. **(08 Marks)**

c. Find the discrete time Fourier transform of the following signals:
   i) $x(n) = (-1)^n U(n)$
   ii) $x(n) = a^{|n|}$. **(06 Marks)**

OR

8   a. State and prove Parseval's theorem with respect to discrete time Fourier transform and indicate the importance of it. **(06 Marks)**

b. The discrete time Fourier transform of a real signal $x(n)$ is $x(\Omega)$. How is the discrete time Fourier transform of the following signals related to $x(\Omega)$.
   i) $y_1(n) = x(-n)$
   ii) $y_3(n) = x(n) * x(-n)$
   iii) $y_2(n) = (1 + \cos n\pi) x(n)$
   iv) $y_4(n) = (-1)^{n/2} x(n)$. **(08 Marks)**

c. Find the continuous time Fourier transform of
   i) $x(t) = \cos(w_o t)$
   ii) $x(t) = \begin{cases} 1; & -T < t < T \\ 0; & \text{otherwise} \end{cases}$ **(06 Marks)**

Module-5

9 a. Find the Z-transform of the following sequences and plot it's ROC.

i) $x(n) = \left(\dfrac{1}{2}\right)^n U(n-2)$

ii) $x(n) = 3\left(\dfrac{-1}{2}\right)^n U(n) - 2[3^n U(-n-1)]$ **(06 Marks)**

b. If x(n) is causal, then prove that

i) $x(0) = \underset{Z \to \infty}{Lt}\, X(z)$

ii) $x(\infty) = \underset{Z \to 1}{Lt}\,[X(Z)(Z-1)]$ **(08 Marks)**

c. A LTI system is given by the system function

$$H(Z) = \dfrac{3 - 4z^{-1}}{(1 - 3.5z^{-1} + 1.5z^{-2})}$$

Specify the ROC of H(z) and h(n) for the following conditions:
i) The system is stable
ii) The system is causal. **(06 Marks)**

OR

10 a. Use the properties of Z-transform to find the Z-transform of the following:

i) $a^{-n}U(-n)$

ii) $\left(\dfrac{1}{2}\right)^n U(n) * \left(\dfrac{1}{3}\right)^n U(n)$

iii) $\left(\dfrac{1}{3}\right)^n U(n) + \left(-\dfrac{1}{2}\right)^n U(n)$ **(08 Marks)**

b. Find the inverse Z-transform of

$X(Z) = \dfrac{Z(Z^2 - 4Z + 5)}{(Z-3)(Z-2)(Z-1)}$ for the ROC $2 < |Z| < 3$ using partial fraction method. **(06 Marks)**

c. Determine the impulse response h(n) and system function H(z) of the system that gives the output

$y(n) = \left(\dfrac{1}{3}\right)^n U(n)$ for an input $x(n) = \left(\dfrac{1}{2}\right)^n U(n) - \dfrac{1}{4}\left(\dfrac{1}{2}\right)^{n-1} U(n-1)$. **(06 Marks)**

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