CBCS SCHEME

USN | | | | | | | | | | 18CS36

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Using truth table, show that the compound propositions $p \wedge (\neg q \vee r)$ and $p \vee (q \wedge \neg r)$ are not logically equivalent. **(06 Marks)**

b. Test the validity of the following argument.
If I study, I will not fail in the examination.
If I do not watch TV in the evening, I will study.
I failed in the examination.
$\therefore$ I must have watched TV in the evenings. **(06 Marks)**

c. Consider the following open statements with set of real numbers as the universe:
$p(x) : x \geq 0$, $q(x) : x^2 \geq 0$, $r(x) : x^2 - 3x - 4 = 0$, $s(x) : x^2 - 3 > 0$
Determine the truth values of the following statements:
(i) $\exists x$, $p(x) \wedge q(x)$ (ii) $\forall x$, $p(x) \rightarrow q(x)$ (iii) $\forall x$, $q(x) \rightarrow s(x)$
(iv) $r(x) \rightarrow p(x)$ **(08 Marks)**

OR

2 a. Prove the following logical equivalences using laws of logic:
(i) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$
(ii) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ **(07 Marks)**

b. Prove that the following argument is valid.
$\forall x$, $[p(x) \vee q(x)]$
$\exists x$, $\neg p(x)$
$\forall x$, $[\neg q(x)\ r(x)]$
$\forall x$, $[s(x) \rightarrow \neg r(x)]$
$\therefore \exists x$, $\neg s(x)$ **(07 Marks)**

c. Find the possible truth values of p, q and r in the following cases:
(i) $p \rightarrow (q \vee r)$ is false (ii) $p \wedge (q \rightarrow r)$ is true **(06 Marks)**

Module-2

3 a. Prove that, for each $n \in z^+$
$$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$ **(06 Marks)**

b. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find (i) the number of bytes, (ii) the number of bytes that begin with 11 and end with 11. (iii) The number of bytes that begin with 11 and do not end with 11 , and (iv) the number of bytes that begin with 11 or end with 11. **(08 Marks)**

c. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these, all four A's are together? How many of them begin with S? **(06 Marks)**

OR

4 a. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways she can invite them in the following situations:
(i) There is no restriction on the choice.
(ii) Two particular persons will not attend separately.
(iii) Two particular persons will not attend together. **(10 Marks)**

b. Find the coefficient of x° in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$ **(05 Marks)**

c. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(05 Marks)**

Module-3

5 a. For any non-empty sets A, B, C prove that following results:
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **(08 Marks)**

b. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5\}$. A function $f : A \to B$ is defined by $F = \{(1, 3), (2, 3), (3, 4), (4, 5), (5, 4)\}$
Find $f^{-1}(B_1)$ and $f^{-1}(B_2)$, where $B_1 = \{3, 4\}$, $B_2 = \{4, 5\}$ **(04 Marks)**

c. Let $s = \{1, 2, 3\}$ and p(s) be the power set of s on p(s), define the relation R by XRY is and only if $X \leq Y$. Show that this relation is a partial order on p(s). Draw its Hasse diagram. **(08 Marks)**

OR

6 a. Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is inverse of f and f is inverse of g. **(06 Marks)**

b. State pigeon hole principle.
ABC is an equilateral triangle, whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm. **(08 Marks)**

c. Define equivalence relation. For the set $A = \{1, 2, 3, 4, 5, 6\}$, consider the partition $P = \{A_1, A_2\}$ where $A_1 = \{1, 3, 5\}$ and $A_2 = \{2, 4, 6\}$. Determine the corresponding equivalence relation R. **(06 Marks)**

Module-4

7 a. The number of virus affected files in a system is 1000 (to start with) and this increases by 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. How many integers between 1 and 300 (inclusive) are (i) divisible by at least one of 5, 6, 8? (ii) divisible by none of 5, 6, 8? **(08 Marks)**

c. Solve the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$, given $a_0 = 1$, $a_1 = 4$, $a_2 = 28$. **(06 Marks)**

OR

8 a. Find the number of derangements of 1, 2, 3, 4. List all the derangements. **(05 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed to 4 boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(09 Marks)**

c. Solve the recurrence relation

$$a_n - 6a_{n-1} + 9a_{n-2} = 0 \quad \text{for } n \geq 2$$

Given that $a_0 = 5$, $a_1 = 12$ **(06 Marks)**

Module-5

9 a. Define the following : Graph, Complement of a graph and Graph isomorphism. **(06 Marks)**

b. Define isomorphism graphs. Determine whether the following graphs are isomorphic or not. [Refer Fig.Q9(b)]

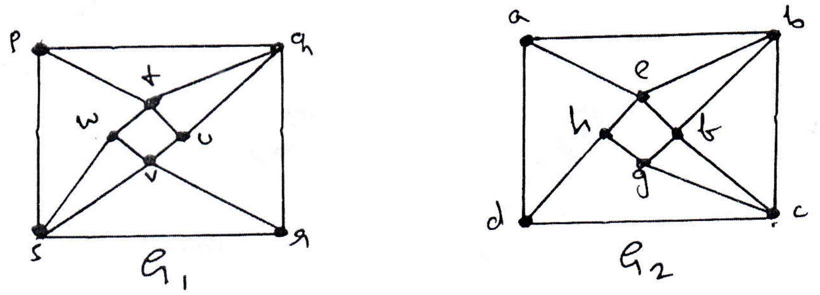

Fig.Q9(b) **(06 Marks)**

c. Apply merge sort to the list

– 9, 6, 5, –3, 4, 2, –7, 8, –5, 10, –11, 0, 1 **(08 Marks)**

OR

10 a. Prove the connected graph with n vertices and n – 1 edges is a tree. **(05 Marks)**

b. Determine the order |V| of the graph G = (V, E) in the following cases :

(i) G is a cubic graph with 9 edges.

(ii) G has 10 edges with 2 vertices of degree 4 and all others of degree 3. **(05 Marks)**

c. Define an optimal prefix code. Obtain the optimal prefix code for the message

MISSION SUCCESSFUL

Indicate the code for the message. **(10 Marks)**

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