CBCS SCHEME

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BMATM201

# Second Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024
## Mathematics – II for ME Stream

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. VTU Formula Hand Book is permitted.*
*3. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$ | 07 | L3 | CO1 |
| | b. | Evaluate $\int_0^{4a} \int_{2\sqrt{ax}}^{x^2/4a} xy\, dy\, dx$ by changing the order of integration. | 07 | L3 | CO1 |
| | c. | Derive the relation $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ | 06 | L2 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | By changing into polar coordinates, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dx\,dy$ | 07 | L3 | CO1 |
| | b. | Using double integration find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ | 07 | L3 | CO1 |
| | c. | Write a modern mathematical tool/program to find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ | 06 | L3 | CO5 |
| | | **Module – 2** | | | |
| Q.3 | a. | Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, –2, 1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$ | 07 | L2 | CO2 |
| | b. | If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at the point (1, –1, 1). | 07 | L2 | CO2 |
| | c. | Define a solenoidal vector. Find the value of 'a' for which $\vec{F}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal. | 06 | L2 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | Find the work done by a force $\vec{F}=(2y-x^2)\hat{i}+6yz\,\hat{j}-8xz^2\,\hat{k}$ from the point (0, 0, 0) to the point (1, 1, 1) along the straight line joining these points. | 07 | L3 | CO2 |
| | b. | Use Stoke's theorem to evaluate $\int_c \vec{F}\cdot d\vec{r}$, where $\vec{F}=(x^2+y^2)\hat{i}-2xy\,\hat{j}$ and C is bounded by $x=\pm a$, $y=0$, $y=b$. | 07 | L3 | CO2 |
| | c. | Using modern mathematical tools write a program to evaluate $\int_c [(x+2y)dx+(x-2y)dy]$, where C is the region bounded by coordinate axes and the lines x = 1 and y = 1, using Green's theorem. | 06 | L2 | CO5 |

## Module – 3

**Q.5** **a.** Form the PDE by eliminating the arbitrary function from $f(x + y + z,\ x^2 + y^2 + z^2) = 0$. (07, L2, CO3)

**b.** Solve $\dfrac{\partial^2 z}{\partial x^2} = xy$ m subject to the conditions $\dfrac{\partial z}{\partial x} = \log(1+y)$, when $x = 1$ and $z = 0$ when $x = 0$. (07, L3, CO3)

**c.** Derive one dimensional wave equation. (06, L2, CO3)

**OR**

**Q.6** **a.** Form the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$ (07, L2, CO3)

**b.** Solve $\dfrac{\partial^2 z}{\partial y^2} = z$ , given that when $y = 0$, $z = e^x$ and $\dfrac{\partial z}{\partial y} = e^{-x}$. (07, L3, CO3)

**c.** Derive one-dimensional heat equation in the standard form as

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(06, L2, CO3)

## Module – 4

**Q.7** **a.** Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ lies between 2 and 3 by the Regula – Falsi method. Carry out four approximations. (07, L2, CO4)

**b.** The area of a circle (A) corresponding to diameter (D) is given below.

| D | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|
| A | 5026 | 5674 | 6362 | 7088 | 7854 |

Find the area corresponding to diameter 105 using an approximate interpolation formula. (07, L2, CO4)

**c.** Use Simpson's 1/3rd rule to find $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$ by taking 6 subintervals. (06, L2, CO4)

**OR**

**Q.8** **a.** Use Newton-Raphson method to find $\sqrt[3]{37}$ correct to 3 decimal points. (07, L1, CO4)

**b.** From the data given in the following table, find the number of students who obtained marks between 40 and 45.

| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| No. of Students | 31 | 42 | 51 | 35 | 31 |

(07, L2, CO4)

**c.** Using Lagrange's interpolation formula, fit a polynomial which passes through the points (–1, 0), (1, 2), (2, 9), (3, 8) and hence estimate the value of y when x = 2.2. (06, L2, CO4)

## Module – 5

**Q.9** **a.** Solve the differential equation $\dfrac{dy}{dx} = x^2 + y^2$, given y(0) = 1 to find the value of y(0.1) by using the Taylor series method taking the terms up to 4th order. (07, L1, CO4)

**b.** Given $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ with the initial condition y = 1 when x = 0. Find approximately y for x = 0.2 by Modified Euler's method. Carry out 3 modifications. Take h = 0.1. (07, L2, CO4)

**c.** Given that $\dfrac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 by applying Milne's method. (06, L2, CO4)

| | | OR | | | |
|---|---|---|---|---|---|
| Q.10 | a. | Find the value of y(0.1) using the R-K method of fourth order, for the given equation $\frac{dy}{dx} = 3e^x + 2y$ , y(0) = 1. | 07 | L2 | CO4 |
| | b. | Use modified Euler's method to find y(0.1) given $\frac{dy}{dx} = 3x + \frac{y}{2}$, y(0) = 1, taking h = 0.1. | 07 | L2 | CO4 |
| | c. | Write modern mathematical tool/program to apply Milne's Predictor-Corrector method to solve $\frac{dy}{dx} = x - y^2$, y(0) = 2 , obtain y(0.8). Take h = 0.2. Use Runge-Kutta method to calculate required initial values. | 06 | L2 | CO5 |

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