

CBCS SCHEME

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BMATE201

Second Semester B.E./B.Tech. Degree Examination, June/July 2023

Mathematics – II for EEE Stream

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. VTU Formula Hand Book is permitted.

3. M : Marks , L: Bloom's level , C: Course outcomes.

Module – 1			M	L	C
Q.1	a.	Find the angle between the direction to the normals to the surface $x^2yz = 1$ at the point $(-1, 1, 1)$ and $(1, -1, -1)$.	7	L1	CO1
	b.	If $\vec{F} = \text{grad}(xy^3z^2)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, -1, 1)$.	7	L2	CO1
	c.	Find the divergence and curl of the vector $\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at the point $(2, -1, 1)$	6	L1	CO1
OR					
Q.2	a.	Using Green's theorem evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.	7	L3	CO1
	b.	Use stoke's theorem for vector $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by $x = 0$, $x = a$, $y = 0$, $y = b$.	7	L3	CO1
	c.	Using modern mathematical tools, write the code to find the gradient of $\phi = x^2y + 2xz - 4$.	6	L3	CO5
Module – 2					
Q.3	a.	Define a subspace. Show that the intersection of any two subspaces of a vector space V is also a subspace of V.	7	L2	CO2
	b.	Show that the set $B = \{(1, 1, 0) (1, 0, 1) (0, 1, 1)\}$ is a basis of the vector space $V_3(\mathbb{R})$.	7	L3	CO2
	c.	Prove that $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(a, b) = (a+b, a-b, b)$ is a linear transform.	6	L3	CO2
OR					
Q.4	a.	Show that the set $S = \{(1, 0, 1) (1, 1, 0) (-1, 0, -1)\}$ is linearly dependent in $V_3(\mathbb{R})$.	7	L3	CO2
	b.	Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x+y, x-y, 2x+z)$. Find the rank and nullity of T and verify rank of T + nullity of T = $\dim(\mathbb{R}^3)$.	7	L2	CO2
	c.	Using the modern mathematical tool, write the code to find the dimension of subspace spanned by the vectors $(1, 2, 3) (2, 3, 1)$ and $(3, 1, 2)$	6	L3	CO5

Module - 3

Q.5	a.	Find the Laplace transform of, (i) $e^{-4t}(2 \cos 6t - 3 \sin 5t)$ (ii) $\frac{\cos 2t - \cos 3t}{t}$	7	L1	CO3
	b.	Find the Laplace transform of a square wave function, $f(t) = \begin{cases} E & 0 \leq t \leq \frac{T}{2} \\ -E & \frac{T}{2} \leq t \leq T \end{cases}$ Show that $L\{f(t)\} = \frac{E}{S} \tanh\left(\frac{ST}{4}\right)$.		L2	CO3
c.		Express $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t \geq 2\pi \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$.	6	L3	CO3

OR

Q.6	a.	Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)(s+3)}\right]$.	7	L1	CO3
	b.	Find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$ using convolution theorem.			
	c.	Solve by Laplace transform method $y'' + 4y' + 4y = e^{-t}$ with $y(0) = y'(0) = 0$			

Module - 4

Q.7	a.	Find the real root of the equation $x \log_{10} x = 1.2$ by using the Regula-Falsi method between 2 and 3 (three iterations).	7	L1	CO4								
	b.	Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, find $\sin 52^\circ$ using Newton's forward interpolation formula.											
	c.	Using Lagrange's interpolation formulae to find $f(5)$ from the following data: <table border="1" style="display: inline-table;"> <tr> <td>x</td> <td>1</td> <td>3</td> <td>4</td> <td>6</td> <td>9</td> </tr> <tr> <td>f(x)</td> <td>-3</td> <td>9</td> <td>30</td> <td>132</td> <td>156</td> </tr> </table>				x	1	3	4	6	9	f(x)	-3
x	1	3	4	6	9								
f(x)	-3	9	30	132	156								

OR

Q.8	a.	Find the real root of the equation, $x \tan x + 1 = 0$ which is near to $x = \pi$ by using Newton-Raphson method.	7	L2	CO4						
	b.	Using Newton's divided difference formulae and find $f(4)$ given the data : <table border="1" style="display: inline-table;"> <tr> <td>x</td> <td>0</td> <td>2</td> <td>3</td> <td>6</td> </tr> <tr> <td>f(x)</td> <td>-4</td> <td>2</td> <td>14</td> <td>158</td> </tr> </table>				x	0	2	3	6	f(x)
x	0	2	3	6							
f(x)	-4	2	14	158							
c.	Evaluate $\int_0^{0.6} e^{-x^2} dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking seven ordinates.										

Module - 5

Q.9	a.	Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ by using Taylor's method upto 4 th degree terms and find the value of $y(1.1)$.	7	L3	CO4
	b.	Using the Runge-Kutta method of order 4 find y at $x = 0.1$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$	7	L3	CO4
	c.	Apply Milne's predictor corrector method, find $y(0.4)$ from $\frac{dy}{dx} = 2e^x y$	6	L2	CO4

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

Q.10	a.	Solve by using modified Euler's method $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ at $x = 0.2$.	7	L3	CO4
	b.	Using the Runge-Kutta method of 4 th order find $y(0.2)$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$.	7	L3	CO4
	c.	Using modern mathematical tools write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at $y(0.1)$. Given that $y(0) = 1$ by Runge Kutta 4 th order.	6	L2	CO5
