

# CBCS SCHEME

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18MAT41

## Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025 Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Show that  $w = f(z) = z + e^z$  is analytic and hence find  $\frac{dw}{dz}$ . (06 Marks)
- b. Derive Cauchy's – Riemann equations in polar form. (07 Marks)
- c. If  $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$  then find analytic function  $f(z) = u + iv$ . (07 Marks)

### OR

- 2 a. Show that the real and imaginary parts of an analytic function  $f(z) = u + iv$  are harmonic. (06 Marks)
- b. If  $f(z)$  is an analytic function then show that  

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4 |f'(z)|^2$$
 (07 Marks)
- c. If  $u = \left( r + \frac{1}{r} \right) \cos \theta$  then find the corresponding analytic function  $f(z) = u + iv$ . (07 Marks)

### Module-2

- 3 a. State and prove Cauchy's integral formula. (06 Marks)
- b. Discuss the conformal transformation  $w = f(z) = z^2$ . (07 Marks)
- c. Find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = i, 0, -i$ . (07 Marks)

### OR

- 4 a. Evaluate  $\int_C z^2 dz$  along the curve made up of two line segments, one from  $z = 0$  to  $z = 3$  and another from  $z = 3$  to  $z = 3 + i$ . (06 Marks)
- b. Evaluate  $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ , where  $C$  is the circle  $|z| = 3$ . (07 Marks)
- c. Find the bilinear transformation which maps the points  $z = -1, 0, 1$  into the points  $w = 0, i, 3i$ . (07 Marks)

### Module-3

- 5 a. The probability distribution of a random variable  $X$  is given by the following table:

$X(=x_i)$	-3	-2	-1	0	1	2	3
$P(X)$	k	2k	3k	4k	3k	2k	k

Find (i) The value of  $k$ , (ii)  $P(x \leq 1)$ , (iii)  $P(-1 < x \leq 2)$

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

- b. The probability that a pen manufactured by a factory be defective is  $1/10$ . If 12 such pens are manufactured, what is the probability that (i) Exactly 2 are defective (ii) Atleast 2 are defective (iii) None of them are defective. (07 Marks)
- c. The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) Ends less than 5 minutes (ii) Between 5 and 10 minutes. (07 Marks)

OR

- 6 a. The probability density function of a random variable X is

$$f(x) = \begin{cases} Kx^2 & , 0 < x < 3 \\ 0 & , \text{otherwise} \end{cases}$$

Find (i) The value of K (ii)  $P(1 < x < 2)$  (iii)  $P(x \leq 1)$  (06 Marks)

- b. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes what is the probability that a shower will last for (i) Ten minutes or more (ii) Less than Ten minutes (iii) Between 10 and 12 minutes. (07 Marks)
- c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65, (ii) more than 75 (iii) 65 to 75. [ $\phi(1) = 0.3413$ ] (07 Marks)

**Module-4**

- 7 a. Compute the rank correlation coefficient for the following data:

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

(06 Marks)

- b. Find a best fitting straight line  $y = ax + b$  for the data below:

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

- c. Obtain the lines of regression and hence find the coefficient of correlation for the data below:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

OR

- 8 a. If  $\theta$  is the acute angle between the lines of regression then show that

$$\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left[ \frac{1 - r^2}{r} \right] \quad (06 \text{ Marks})$$

- b. Find a best fitting second degree parabola of the form  $y = ax^2 + bx + c$  for the data below:

x	1	2	3	4	5
y	10	12	13	16	19

(07 Marks)

- c. Find the coefficient of correlation for the following data:

x	10	14	18	22	26	30
y	18	12	24	06	30	36

(07 Marks)

**Module-5**

- 9 a. The joint probability of discrete random variables X and Y is given below:

	Y	1	3	9
X				
2		1/8	1/24	1/12
4		1/4	1/4	0
6		1/8	1/24	1/12

Determine (i) Marginal distribution of X and Y. (ii) Covariance and correlation of X and Y. (06 Marks)

- b. A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800, it was revealed that 180 families were illiterates. Find the probable limits of the illiterates families in the population of 2000 at 1% level of significance. (07 Marks)
- c. A group of 10 boys fed at diet A and another group of 08 boys fed on another diet B for a period of 06 months record the following increase in weights in pounds.

Diet A	05	06	08	01	12	04	03	09	06	10
Diet B	02	03	06	08	10	01	02	08	-	-

Test whether diet A and B differ significantly regarding their effect on increase in weight.  $[t_{0.05} = 2.12]$  (07 Marks)

**OR**

- 10 a. Explain the terms :  
 (i) Null hypothesis  
 (ii) Type-I and Type-II errors  
 (iii) Level of significance. (06 Marks)

- b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4  
 Can it be concluded that the stimulus will increase the blood pressure?  $[t_{0.05} = 2.201]$  (07 Marks)

- c. A sample analysis of examination, result of 500 students was made, it was found that 220 students had failed, 170 had secured third class, 90 had secured second class, 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories?  $[\chi^2_{0.05} = 7.81]$ . (07 Marks)

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