Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing to echelon form. (06 Marks)
 - b. Solve the system of equations by Gauss elimination method:

$$x + y + z = 9$$

 $x - 2y + 3z = 8$
 $2x + y - z = 3$

(07 Marks)

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (07 Marks)

OR

2 a. Find the rank of the following matrix by applying elementary row transformation

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(06 Marks)

b. Solve the following system of linear equations by Gauss elimination method:

$$x + 2y + z = 3$$
, $2x + 3y + 3z = 10$, $3x - y + 2z = 13$

(07 Marks)

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (07 Marks)

Module-2

3 a. A function f(x) is given by the following table

X	0	\1	2	3.	4	5	6
f(x)	176	185	194	203	212	220	229
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Obtain the value of f(x) at x = 0.6 by using appropriate interpolation formula.

(06 Marks)

- b. The equation $x^3 3x + 4 = 0$ has one real root between -2 and -3. Find the root to three places of decimals by using Regula-Falsi method. (07 Marks)
- c. Using Simpson's $1/3^{rd}$ rule, evaluate $\int_{0}^{\infty} e^{-x^{2}}$ by dividing the interval (0, 1) into 10 sub intervals, (h = 0.1).

OR

- 4 a. Find f(2.5) by using Newton's backward interpolation formula given that f(0) = 7.4720, f(1) = 7.5854, f(2) = 7.6922, f(3) = 7.8119, f(4) = 7.9252. (06 Marks)
 - b. Find the real root of the equation $xe^x 2 = 0$, correct to three decimal places by using Newton Raphson method. (07 Marks)
 - c. Evaluate $\int_{0}^{1} \frac{x \, dx}{1 + x^2}$ by Weddle's rule taking seven ordinates. (07 Marks)

Module-3

5 a. Solve:
$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$
 (06 Marks)

b. Solve:
$$(D^2 + 7D + 12)y = \cosh x$$
 (07 Marks)

c. Solve:
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 2x$$
 (07 Marks)

OR

6 a. Solve:
$$(D^3 - 4D^2 + 5D - 2)y = 0$$
 (06 Marks)

b. Solve:
$$\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$$
 (07 Marks)

c. Solve:
$$(D^2 - 4D + 3)y = \sin 3x \cdot \cos 2x$$
 (07 Marks)

Module-4

- 7 a. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' $z = (x^2 + a) (y^2 + b)$ (06 Marks)
 - b. Form the partial differential equation by eliminating arbitrary functions "f" from $z = f\left(\frac{xy}{z}\right)$.

 (07 Marks)
 - c. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \sin y$, given that $\frac{\partial z}{\partial y} = -2 \sin y$ when x = 0 and z = 0 when y is an odd multiple of $\pi/2$.

OR

- 8 a. Form the partial differential equation by eliminating arbitrary function 'f' from the function $f(xy + z^2, x + y + z) = 0$ (06 Marks)
 - b. Form partial differential equation by eliminating arbitrary functions 'f' and 'g' from the function z = y f(x) + x g(y) (07 Marks)
 - c. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. (07 Marks)

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Module-5

- 9 a. A bag contains 8-white and 6-red balls. Find the probability of drawing two balls of the same colour. (06 Marks)
 - b. Three machines A, B, C produces 50%, 30%, 20% of the items in a factory. The percentage of defective outputs are 3, 4, 5. If an item is selected at random, what is the probability that it is defective? What is the probability that it is for A? (07 Marks)
 - c. A can hit a target 3-times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that i) two shots hit ii) at least two shots hit?

 (07 Marks)

OR

10 a. State and prove Baye's theorem.

(06 Marks)

- b. State the axiomatic definition of probability. For any two arbitrary events A and B, prove that $P(A \cup B) = P(A) + P(B) P(A \cap B)$. (07 Marks)
- c. If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$. Then find P(A/B), P(B/A), $P(\overline{A/B})$, $P(\overline{B/A})$ and P(A/B). (07 Marks)
