

--	--	--	--	--	--	--	--	--	--

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (06 Marks)
- b. A periodic function of period $\frac{2\pi}{\omega}$ is defined by, $f(t) = \begin{cases} E \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ (07 Marks)
- c. Solve : $y'' - y' - 2y = 0$; given $y(0) = 0$ and $y'(0) = 6$ by Laplace transformation method. (07 Marks)

OR

- 2 a. Find $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$. (06 Marks)
- b. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$. (07 Marks)
- c. Using unit step function, find the Laplaces transform of, $f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ \sin 2t, & \pi \leq t \leq 2\pi \\ \sin 3t, & t \geq 2\pi \end{cases}$. (07 Marks)

Module-2

- 3 a. Obtain the Fourier Series for the function, $f(x) = x^2$, $0 < x < 2\pi$. (06 Marks)
- b. Find the Fourier series of $f(x)$,
Where $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. (07 Marks)
- c. Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (07 Marks)

OR

- 4 a. Obtain Fourier series for the function, $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$. (06 Marks)
- b. Find the Fourier half-range cosine series of the function $f(x) = (x+1)$, in $(0, 1)$. (06 Marks)
- c. Compute the first harmonic of the Fourier series of $f(x)$ given in the following table :

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

Module-3

- 5 a. Find the z-transform of, $3n - 4 \sin\left(\frac{n\pi}{4}\right) + 5a$. (06 Marks)
- b. Compute the inverse z - transform of, $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (07 Marks)
- c. Find the Fourier transform of, $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$, Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. (07 Marks)

OR

- 6 a. Find the Fourier sine transform of e^{-ax} . (06 Marks)
- b. If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate u_2 and u_3 . (07 Marks)
- c. Using the z-transform, solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0$, $u_1 = 1$. (07 Marks)

Module-4

- 7 a. Find by Taylor's series method the value of y at $x = 0.1$ and $x = 0.2$ to four places of decimals from,
 $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$. (06 Marks)
- b. Apply Runge-Kutta fourth order method to find an approximate value of y when $x = 0.2$, given $\frac{dy}{dx} = x + y$, $y(0) = 1$. (07 Marks)
- c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3) = 2.09$, find $y(0.4)$ by employing the Milne's predictor-corrector formula, use corrector formula twice. (07 Marks)

OR

- 8 a. Using modified Euler's method, solve the IVP $\frac{dy}{dx} = x + \sqrt{y}$, $y(0) = 1$ at $x = 0.2$, perform three modifications. (06 Marks)
- b. Using the fourth order Runge-Kutta method, solve the IVP $\frac{dy}{dx} = \frac{1}{x+y}$ at the point $x = 0.5$. Given that $y(0.4) = 1$. (07 Marks)
- c. Given $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Determine $y(1.4)$ by Adams-Bashforth method. (07 Marks)

Module-5

- 9 a. Using Runge-Kutta method of fourth order solve the differential equation,
 $\frac{d^2y}{dx^2} = x \left(\frac{dy}{dx} \right)^2 - y^2$, with $y(0) = 1$, $y'(0) = 0$ at $x = 0.2$. (06 Marks)
- b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (07 Marks)

- c. On which curve the functional,

$$\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy) dy, y(0) = 0, y\left(\frac{\pi}{2}\right) = 0 \text{ be extremized.}$$

(07 Marks)

OR

- 10 a. Apply Milne's method to compute $y(0.8)$, given that $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}$ and

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(06 Marks)

- b. Prove that the geodesics on a plane are straight line.

(07 Marks)

- c. Find the extremal of the functional, $I = \int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 4y \cos x) dx$, given that $y(0) = 0$,

$$y\left(\frac{\pi}{2}\right) = 0.$$

(07 Marks)

* * * * *