

CBCS SCHEME

USN

BMATE201

Second Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024 Mathematics-II for EEE Stream

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. VTU Formula Hand Book is permitted.

3. M : Marks , L: Bloom's level , C: Course outcomes.

Module - 1			M	L	C
Q.1	a.	Find the directional derivative of $\phi = xy^3 + yz^3$ at $(2, -1, 1)$ along $i + 2j + 2k$.	7	L2	CO1
	b.	Find div \vec{F} and curl \vec{F} , if $\vec{F} = y^3 z^2 \hat{i} + 3xy^2 z^2 \hat{j} + 2xy^3 z \hat{k}$ at $(1, -1, 1)$.	7	L2	CO1
	c.	If $\vec{V} = 3xy^2 z^2 \hat{i} + y^3 z^2 \hat{j} + 2y^2 z^3 \hat{k}$ and $\vec{F} = (x^2 - yz) \hat{i} + (y^2 - zx) \hat{j} + (z^2 - xy) \hat{k}$, show that \vec{V} is solenoidal and \vec{F} is irrotational.	6	L2	CO1
OR					
Q.2	a.	Find the workdone in moving a particle by the force field $\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + z \hat{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.	7	L2	CO1
	b.	Using Green's theorem evaluate $\int (xy - x^2) dx + x^2 y dy$ over the region bounded by $y = x$ and $y = x^2$.	7	L3	CO1
	c.	Write the mathematical tool program to find the divergence of the vector field. $\vec{F} = x^2 y z \hat{i} + y^2 z x \hat{j} + z^2 x y \hat{k}$.	6	L3	CO5
Module - 2					
Q.3	a.	If W is the set of all points in R^3 satisfying the equation $a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$, then prove that W is a subspace of R^3 .	7	L2	CO2
	b.	Prove that in $V_3(R)$, the vectors $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ are linearly independent.	7	L2	CO2
	c.	Prove that the transformation $T : R^3 \rightarrow R^3$ defined by $T(x_1, x_2, x_3) = (0, x_2, x_3)$ is linear.	6	L2	CO2
OR					
Q.4	a.	Express the vector $(2, -1, -8)$ as a linear combination of the vectors $(1, 2, 1)$, $(1, 1, -1)$ and $(4, 5, -2)$.	7	L2	CO2
	b.	Find the matrix of the linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (x + y, y + z)$ relative to the bases $B_1 = \{(1, 1, 0), (1, 0, 1), (1, 1, -1)\}$, $B_2 = \{(2, -3), (1, 4)\}$.	7	L2	CO2

	c.	Using the modern mathematical tool. Write the code to represent the reflection transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and to find the image of vector $(10, 0)$ when it is reflected about the y-axis.	6	L3	CO5
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Module - 3

Q.5	a.	Find the Laplace transform of i) $te^{-t}\sin 3t$ ii) $\frac{e^{at}-e^{bt}}{t}$	7	L2	CO3
	b.	Find the Laplace transform of the square wave function of period 'a' defined by $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$	7	L3	CO3
	c.	Express $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4 \\ 6, & t > 4 \end{cases}$ in terms of unit step function and hence find its Laplace transform.	6	L3	CO3

OR

Q.6	a.	Find $L^{-1}\left\{\frac{2s-1}{(s-1)(s-3)}\right\}$.	7	L2	CO3
	b.	Find $L^{-1}\left\{\frac{1}{s(s^2 + a^2)}\right\}$ using convolution theorem.	7	L2	CO3
	c.	Solve by Laplace transform $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 0, y(0) = y'(0) = 3$.	6	L3	CO3

Module - 4

Q.7	a.	Find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$ by Newton-Raphson method.	7	L2	CO4										
	b.	Using Newton's forward interpolation formula find y at $x = 5$ for the data: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr> <tr> <td>y</td><td>1</td><td>3</td><td>8</td><td>16</td></tr> </table>	x	4	6	8	10	y	1	3	8	16	7	L3	CO4
x	4	6	8	10											
y	1	3	8	16											
	c.	Find the interpolating polynomial using Newton's divided difference formula for the data. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr> <tr> <td>y</td><td>2</td><td>3</td><td>12</td><td>147</td></tr> </table>	x	0	1	2	5	y	2	3	12	147	6	L2	CO4
x	0	1	2	5											
y	2	3	12	147											

OR

Q.8	a.	Find the real root of the equation $x \log_{10}x = 1.2$ by Regula Falsi method between 2 and 3 (three iterations).	7	L2	CO4
	b.	Find y at $x = 5$, if $y(1) = +3, y(3) = 9, y(4) = 30, y(6) = 132$ using Lagrange's interpolation formula.	7	L2	CO4

	c.	Evaluate $\int_0^{\pi/2} \sqrt{\cos x} dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking 6 equal parts.	6	L3	CO4
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Module - 5

Q.9	a.	Use Taylor's series method to find $y(0.1)$ by considering upto 3 rd degree term, given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$.	7	L3	CO4
	b.	Given $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$, $h = 0.1$ using Runge-Kutta method of 4 th order find y at $x = 0.1$.	7	L3	CO4
	c.	Apply Milne's predictor-corrector method, find $y(0.4)$ given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, $y(0.1) = 1.1113$, $y(0.2) = 1.2507$, $y(0.3) = 1.426$.	6	L2	CO4

OR

Q.10	a.	Using modified Euler's method, find $y(0.1)$ given that $\frac{dy}{dx} = x^2 + y$ and $y(0) = 1$, take $h = 0.05$ and perform two iterations in each stage.	7	L2	CO4
	b.	Apply Milne's method to find $y(4.4)$ given that $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0142$ and $\frac{dy}{dx} = \frac{2-y^2}{5x}$	7	L2	CO4
	c.	Write a modern mathematical tool program to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, $h = 0.1$ using R - K method of 4 th order.	6	L3	CO5
