CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. If $2\cos\theta = x + \frac{1}{x}$, show that $2\cos n\theta = x^n + \frac{1}{x^n}$ and $2i\sin n\theta = x^n - \frac{1}{x^n}$. Also show that $\frac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$. **(08 Marks)**

b. Find the real part of $\frac{1}{1+\cos\theta+\sin\theta}$. **(06 Marks)**

c. Express $\left(\frac{3+4i}{3-4i}\right)$ in a + ib form. **(06 Marks)**

OR

2 a. If $\vec{a} = 2i+3j-4k$ and $\vec{b} = 8i-4j-k$ prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a}\times\vec{b}|$. **(08 Marks)**

b. If $\vec{a} = 3i-2j-4k$ and $\vec{b} = i+j-2k$, find :

i) $|2\vec{a}+3\vec{b}|$ ii) $|\vec{a}\cdot\vec{b}|$ iii) angle between $\vec{a}$ and $\vec{b}$. **(06 Marks)**

c. Find a unit vector normal to both the vectors 4i – j + 3k and –2i + j – 2k. Find also the sine of the angle between them. **(06 Marks)**

Module-2

3 a. Obtain the Maclaurin's series expansion of $\sqrt{1+\sin 2x}$ up to the form containing x^4. **(08 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

c. If $u = f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(06 Marks)**

OR

4 a. Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \text{-------}$ by using Macluarin's series notation. **(08 Marks)**

b. If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$ **(06 Marks)**

c. If $u = x + y$ and $v = \frac{y}{x+y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **(06 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the velocity and acceleration at t= 1 in the direction $i - 3j + 2k$. (08 Marks)

b. Find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point $(1, -1, 2)$. (06 Marks)

c. Prove that $\vec{d} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is irrotational. (06 Marks)

OR

6 a. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (08 Marks)

b. If $\vec{A} = xyi + y^2zj + z^2yk$, find $\text{curl}(\text{crul}\,\vec{A})$. (06 Marks)

c. Find 'a' if the vector field $\vec{F} = (ax + 3y + 4z)i + (x - 2y + 3z)j + (3x + 2y - z)k$ is Solenoidal. (06 Marks)

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx \quad (n > 0)$. (08 Marks)

b. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^4}\,dx$. (06 Marks)

c. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dy\,dx$. (06 Marks)

OR

8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, where n is a positive integer. (08 Marks)

b. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x\,dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz\ dx\,dy\,dz$. (06 Marks)

Module-5

9 a. Solve $[(\cos x \cdot \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$ (08 Marks)

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$. (06 Marks)

c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. (06 Marks)

OR

10 a. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ (08 Marks)

b. Solve $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$. (06 Marks)

c. Solve $x\frac{dy}{dx} + y = x^3y^6$. (06 Marks)