

Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025 Mathematics – II for Mechanical Engineering Stream

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. VTU Formula Hand Book is permitted.

3. M : Marks , L: Bloom's level , C: Course outcomes.

Module – 1			M	L	C
Q.1	a.	Evaluate $\int_{-1}^1 \int_0^{2-x} \int_{x-2}^{x+2} (x+y+z) dy dx dz$.	7	L3	CO1
	b.	Evaluate $\int_0^1 \int_{\sqrt{y}}^1 dx dy$ by changing the order of integration.	7	L3	CO1
	c.	With usual notation show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6	L2	CO1
OR					
Q.2	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y dx dy$.	7	L3	CO1
	b.	Evaluate $\iint xy(x+y) dy dx$ taken over the area between $y = x^2$ and $y = x$.	7	L2	CO1
	c.	Write a modern mathematical program to evaluate the integral $\int_0^{3-3x} \int_0^{3-x-y} \int_0^{3-x-y} (xyz) dz dy dx$.	6	L3	CO5
Module – 2					
Q.3	a.	Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1).	7	L2	CO2
	b.	If $\vec{F} = \nabla(xy^3z^2)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point (1, -1, 1).	7	L2	CO2
	c.	Define solenoidal vector. Find the values of a, b, c such that $\vec{F} = (x + y + az)\mathbf{i} + (bx + 2y - z)\mathbf{j} + (x + cy + 2z)\mathbf{k}$ is irrotational.	6	L2	CO2
OR					
Q.4	a.	Apply Green's theorem to evaluate $\int (3x - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $x = 0$, $y = 0$, $x + y = 1$.	7	L3	CO2

	b.	Use Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by $x = 0$, $x = a$, $y = 0$ and $y = b$.	7	L3	CO2														
	c.	Write the modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz\vec{i} + y^2zx\vec{j} + z^2xy\vec{k}$.	6	L3	CO5														
Module – 3																			
Q.5	a.	Form the PDE by eliminating the arbitrary function from the relation $f(x + y + z, x^2 + y^2 - z^2) = 0$.	7	L2	CO3														
	b.	Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y}$, subject to the conditions $\frac{\partial z}{\partial x} = \log_e x$ when $y = 1$ and $z = 0$ when $x = 1$.	7	L3	CO3														
	c.	Derive one-dimensional heat equation.	6	L2	CO3														
OR																			
Q.6	a.	Form the PDE by eliminating the arbitrary function from the relation $z = e^y f(x + y)$.	7	L2	CO3														
	b.	Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.	7	L3	CO3														
	c.	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ using Lagrange's multiplier.	6	L3	CO3														
Module – 4																			
Q.7	a.	Find a real root of $x^3 - 9x + 1 = 0$ in $(2, 3)$ by the Regula-Falsi method in three iterations.	7	L3	CO4														
	b.	Use an appropriate interpolation formula to compute $f(42)$ using the following data: <table><tr><td>x</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>f(x)</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr></table>	x	40	50	60	70	80	90	f(x)	184	204	226	250	276	304	7	L3	CO4
x	40	50	60	70	80	90													
f(x)	184	204	226	250	276	304													
	c.	Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^{0.6} e^{-x^2} dx$ by considering seven ordinates.	6	L3	CO4														
OR																			
Q.8	a.	Find a real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$ correct to four decimal places using Newtons-Raphson method.	7	L3	CO4														
	b.	Using Lagrange's interpolation formula. Find $f(5)$ from the following data : <table><tr><td>x</td><td>1</td><td>3</td><td>4</td><td>6</td><td>9</td></tr><tr><td>f(x)</td><td>3</td><td>9</td><td>30</td><td>132</td><td>156</td></tr></table>	x	1	3	4	6	9	f(x)	3	9	30	132	156	7	L3	CO4		
x	1	3	4	6	9														
f(x)	3	9	30	132	156														

	c.	Use Simpson's $\frac{3}{8}$ rule to obtain the approximate value of $\int_0^{0.3} (1-8x^3)^{\frac{1}{2}} dx$, by considering 3 equal intervals.	6	L3	CO4
Module – 5					
Q.9	a.	Use Taylor's method to find $x = 0.1$ considering terms up to the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$.	7	L2	CO4
	b.	Using Runge-Kutta method of order 4, find y at $x = 0.1$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 1$.	7	L3	CO4
	c.	Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at $x = 0.8$ by applying Milne's method.	6	L3	CO4
OR					
Q.10	a.	Using modified Euler's method, find y at $x = 0.2$ given that $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. Perform three iterations.	7	L3	CO4
	b.	Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation, $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$.	7	L3	CO4
	c.	Using modern mathematical tools write a program to solve $\frac{dy}{dx} = 1 + \left(\frac{y}{x}\right)$ at $y(2)$ taking $h = 0.2$. Given that $y(1) = 2$ by Runge-Kutta method.	6	L3	CO5
