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BMATE201

## Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

### Mathematics – II for EEE stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*  
 2. VTU Formula Hand Book is permitted.  
 3. M : Marks , L: Bloom's level , C: Course outcomes.

Module – 1				M	L	C
Q.1	a.	If $\vec{F} = \nabla(xy^3z^2)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the points (1, -1, 1).		7	L2	CO1
	b.	Show that $\vec{F} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$ is irrotational. Also find a scalar function $\phi$ such that $\vec{F} = \nabla\phi$ .		7	L3	CO1
	c.	Using Green's theorem, evaluate $\oint_C (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$ .		6	L3	CO1
OR						
Q.2	a.	Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ .		7	L2	CO1
	b.	If $\vec{F} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ , evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve represented by $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$ .		7	L2	CO1
	c.	Using modern mathematical tools, write the code to find gradient of $\phi = x^2y + 2xz - 4$ .		6	L3	CO5
Module – 2						
Q.3	a.	Define Vector space, Subspace and Linear dependent.		7	L2	CO2
	b.	Find the dimension and basis of the subspace spanned by the vectors, (2, 4, 2), (1, -1, 0), (1, 2, 1) and (0, 3, 1) in $V_3R$ .		7	L2	CO2
	c.	Prove that the following functions are linear transformation, $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (y, -x, -z)$ .		6	L2	CO2
OR						
Q.4	a.	Determine whether the vectors $V_1 = (1, 2, 3)$ , $V_2 = (3, 1, 7)$ and $V_3 = (2, 5, 8)$ are linearly dependent or linearly independent.		7	L2	CO2
	b.	Let $T: V \rightarrow W$ be a linear transformation defined by, $T(x, y, z) = (x+y, x-y, 2x+z)$ . Find the range, null space, rank, nullity and hence verify the rank nullity theorem.		7	L2	CO2

	c.	Using the modern mathematical tool, write a code to verify whether the following vectors (2, 1, 5, 4) and (3, 4, 7, 8) are orthogonal.	6	L3	CO5												
<b>Module – 3</b>																	
Q.5	a.	Find the Laplace transform of, $2^t + \frac{\cos 2t - \cos 3t}{t} + t \sin t$ .	7	L2	CO3												
	b.	Given $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ where $f(t+a) = f(t)$ , show that $L[f(t)] = \frac{E}{S} \tanh\left(\frac{aS}{4}\right)$ .	7	L3	CO3												
	c.	Express $f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ \sin 2t, & \pi \leq t < 2\pi \\ \sin 3t, & t \geq 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find $L[f(t)]$ .	6	L3	CO3												
<b>OR</b>																	
Q.6	a.	Find the inverse Laplace transform of, $\frac{1}{s(s+1)(s+2)(s+3)}$ .	7	L2	CO3												
	b.	Find $L^{-1}\left[\frac{S}{(S^2 + a^2)^2}\right]$ using convolution theorem.	7	L3	CO3												
	c.	Employ Laplace transform to solve the equation : $y'' + 5y' + 6y = 5e^{2t}$ , $y(0) = 2$ , $y'(0) = 1$ .	6	L3	CO3												
<b>Module – 4</b>																	
Q.7	a.	Using Newton-Raphson method, find the root that lies near $x = 4.5$ of the equation $\tan x = x$ . Correct to four decimal places.	7	L2	CO4												
	b.	The area of a circle (A) corresponding to diameter (D) is given below, <table border="1" data-bbox="370 1509 883 1581"> <tr> <td>D</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr> <tr> <td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr> </table> Find the area corresponding to diameter 105 using an appropriate interpolation formula.	D	80	85	90	95	100	A	5026	5674	6362	7088	7854	7	L3	CO4
D	80	85	90	95	100												
A	5026	5674	6362	7088	7854												
	c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule taking four equal parts.	6	L3	CO4												
<b>OR</b>																	
Q.8	a.	Find the real root of $x \log_{10} x - 1.2 = 0$ by the method of False position. Carry out three iterations.	7	L2	CO4												

	b.	Use Lagrange's interpolation formula to find y at x = 10 given, <table><tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr><tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr></table>	x	5	6	9	11	y	12	13	14	16	7	L2	CO4
x	5	6	9	11											
y	12	13	14	16											
	c.	Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}$ rule.	6	L3	CO4										
Module – 5															
Q.9	a.	Use Taylor's series method to find y at x = 0.1 considering upto the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.	7	L2	CO4										
	b.	Given $\frac{dy}{dx} = 3x + \frac{y}{2}$ , y(0) = 1. Compute y(0.2) by taking h = 0.2, using Runge-Kutta method of fourth order.	7	L2	CO4										
	c.	Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 by using Milne's method.	6	L2	CO4										
OR															
Q.10	a.	Using modified Euler's formula, compute y at x = 0.2 given that $\frac{dy}{dx} = x + y$ , y(0) = 1 and h = 0.2	7	L3	CO4										
	b.	Use fourth order Runge-Kutta method to compute y(1.1) given that $\frac{dy}{dx} = xy^{\frac{1}{3}}$ , y(1) = 1 and h = 0.1	7	L2	CO4										
	c.	Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at y(2) taking h = 0.2 and y(1) = 2 by Runge-Kutta method of order four.	6	L3	CO5										

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