

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025 Mathematics-I for ME Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.*

Module – 1				M	L	C
Q.1	a.	Prove with usual notations, $\tan \phi = r \cdot \frac{d\theta}{dr}$		06	L2	CO1
	b.	Find the angle between the curves, $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$		07	L2	CO1
	c.	Find the pedal equation to the curves $r^n = a^n \cos n\theta$		07	L2	CO1
OR						
Q.2	a.	Prove that for the radius of curvature in Cartesian form $\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$		07	L2	CO1
	b.	Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it.		08	L2	CO1
	c.	Using modern mathematical tools write the code to find the radius of curvature $r = 4(1 + \cos t)$ at $t = \pi/2$		05	L3	CO5
Module – 2						
Q.3	a.	Expand $e^{\sin x}$ by Maclaurin's series upto the terms containing x^4 .		06	L2	CO2
	b.	Evaluate : $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{1/x}$		07	L2	CO2
	c.	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$		07	L2	CO2
OR						
Q.4	a.	If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$		07	L2	CO2
	b.	If $u = \frac{yz}{z}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$		08	L2	CO2
	c.	Using modern mathematical tools, write the code to solve : $y'' - 5y' + 6y = \cos 4x$		05	L3	CO5
Module – 3						
Q.5	a.	Solve : $xy(1 + xy^2) \frac{dy}{dx} = 1$		06	L2	CO3
	b.	Solve : $(x^2 + y^2 + x)dx + xy dy = 0$		07	L2	CO3
	c.	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is the parameter.		07	L2	CO3

OR						
Q.6	a.	Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$	06	L2	CO3	
	b.	Solve the equation $(px - y)(py + x) = 2$ by reducing in to Clairaut's form, taking the substitution. $X = x^2, Y = y^2$.	07	L2	CO3	
	c.	If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C.	07	L3	CO3	
Module - 4						
Q.7	a.	Solve : $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$	06	L2	CO3	
	b.	Solve : $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$	07	L2	CO3	
	c.	Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$	07	L2	CO3	
OR						
Q.8	a.	Solve : $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters.	06	L2	CO3	
	b.	Solve : $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1 + x)^2$, using Cauchy's equation.	07	L3	CO3	
	c.	Solve the Legendre's linear equation $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin \log(1 + x)$	07	L3	CO3	
Module - 5						
Q.9	a.	Find the rank of a matrix by elementary row transformation $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$	06	L2	CO4	
	b.	Investigate the values of λ and μ such that the system of equations: $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$	07	L2	CO4	
	c.	Solve the following system of equations by Gauss-Jordan method. $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$	07	L2	CO4	
OR						
Q.10	a.	Using Rayleigh's power method, find numerically the largest eigen value and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	07	L2	CO4	
	b.	Solve the following system of equations by Gauss-Seidel method. $10x + y + z = 12$; $x + 10y + z = 12$; $x + y + 10z = 12$	08	L2	CO4	
	c.	Using modern mathematical tools, write the code to check whether the following system of homogeneous linear equation has non-trivial solution: $x_1 + 2x_2 - x_3 = 0$; $2x_1 + x_2 + 4x_3 = 0$; $3x_1 + 3x_2 + 4x_3 = 0$	05	L3	CO5	
