# CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐ **BMATS201**

## Second Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024

## Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. VTU Formula Hand Book is permitted.*
*3. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | **Module – 1** | **M** | **L** | **C** |
|---|---|---|---|---|---|
| **Q.1** | **a.** | Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dydxdz$. | 7 | L2 | CO1 |
| | **b.** | Evaluate $\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates. | 7 | L3 | CO1 |
| | **c.** | Show that $\beta(m,n)=\frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$ | 6 | L2 | CO1 |
| | | **OR** | | | |
| **Q.2** | **a.** | Evaluate $\int_{0}^{1}\int_{y}^{\sqrt{y}}(x^2y+xy^2)dxdy$ by changing the order of integration. | 7 | L3 | CO1 |
| | **b.** | Show that $\int_{0}^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_{0}^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$ | 7 | L2 | CO1 |
| | **c.** | Using mathematical tools, write the code to find the area of an ellipse by double integration $A=4\int_{0}^{a}\int_{0}^{\frac{b}{a}\sqrt{a^2-x^2}}dydx$, taking a = 4, b = 6. | 6 | L3 | CO5 |
| | | **Module – 2** | | | |
| **Q.3** | **a.** | Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at (2, -1, 2) along vector 2i – 3j + 6k. | 7 | L2 | CO2 |
| | **b.** | Show that the vector $\vec{F}=\frac{xi+yi}{x^2+y^2}$ is both solenoidal and irrotational. | 7 | L2 | CO2 |
| | **c.** | Prove that the spherical coordinate system is orthogonal. | 6 | L3 | CO2 |
| | | **OR** | | | |
| **Q.4** | **a.** | Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z^2+y^2-x=3$ at (2, -1, 2). | 7 | L2 | CO2 |
| | **b.** | Express the vector $\vec{A}=z\hat{i}-2x\hat{j}+y\hat{k}$ in cylindrical coordinates. | 7 | L2 | CO2 |
| | **c.** | Using mathematical tools, write the code to find the curl of $\vec{F}=x^2yz\hat{i}+y^2zx\hat{j}+z^2xy\hat{k}$. | 6 | L3 | CO5 |

<table>
<tr><td colspan="6"><b>Module – 3</b></td></tr>
<tr><td><b>Q.5</b></td><td><b>a.</b></td><td>Prove that the subset $W = \{(x, y, z) : ax + by + cz = 0; x, y, z \in R\}$ of the vector space $R^3$ is a subspace of $R^3$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td><b>b.</b></td><td>Determine the following vectors are linearly independent or not, $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$ and $x_3 = (1, 2, 2)$ in $R^3$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td><b>c.</b></td><td>Show that the function $T : R^2 \rightarrow R^3$ given by $T(x, y) = (x + y, x - y, y)$ is a linear transformation.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6"><b>OR</b></td></tr>
<tr><td><b>Q.6</b></td><td><b>a.</b></td><td>Determine whether the vectors $v_1 = (1, 2, 3)$, $v_2 = (3, 1, 7)$ and $v_3 = (2, 5, 8)$ are linearly dependent or linearly independent.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td><b>b.</b></td><td>Verify the Rank-Nullity theorem for the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td><b>c.</b></td><td>Consider the vectors $u = (1, 2, 4)$, $v = (2, -3, 5)$, $w = (4, 2, -3)$ in $R^3$. Find:<br>i) $<u.v>$ ii) $<u.w>$ iii) $<v.w>$ iv) $<(u + v).w>$</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6"><b>Module – 4</b></td></tr>
<tr><td><b>Q.7</b></td><td><b>a.</b></td><td>Find an approximate value of the root of the equation $x^3 - x^2 - 1 = 0$, using the Regula-Falsi method upto four decimal places of accurancy, where root lies between 1.4 and 1.5.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>b.</b></td><td>Using Newton's divided difference formula evaluate f(4) from the following:
<table>
<tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr>
<tr><td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr>
</table></td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>c.</b></td><td>Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Trapezoidal rule by taking 7 ordinates.</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6"><b>OR</b></td></tr>
<tr><td><b>Q.8</b></td><td><b>a.</b></td><td>Find an approximate root of the equation $x \log_{10} x - 1.2 = 0$ corrected to five decimal places where root lies near 2.5 by Newton-Raphson method.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>b.</b></td><td>The area A of a circle of diameter d is given for the following values. Calculate the area of a circle of diameter 82 by using Newton's forward interpolation formula.
<table>
<tr><td>d</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr>
<tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr>
</table></td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>c.</b></td><td>Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.</td><td>6</td><td>L2</td><td>CO4</td></tr>
</table>

| | | | | | |
|---|---|---|---|---|---|
| | | **Module – 5** | | | |
| **Q.9** | **a.** | Find by Taylor's series method the value of y at x = 0.1 to five places of decimals from $\frac{dy}{dx} = x^2y - 1$ with an initial condition y(0) = 1. | **7** | **L2** | **CO4** |
| | **b.** | Using the Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2 taking h = 0.2. | **7** | **L2** | **CO4** |
| | **c.** | Given that $\frac{dy}{dx} = x^2(1 + y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979. Compute y at x = 1.4 by applying Milne's method. | **6** | **L2** | **CO4** |
| | | **OR** | | | |
| **Q.10** | **a.** | Using modified Euler's method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ at x = 0.1 corrected to four decimal places by taking h = 0.1, with initial condition y(0) = 1. | **7** | **L2** | **CO4** |
| | **b.** | Given that $\frac{dy}{dx} = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y(0.8) by Milne's method. | **7** | **L2** | **CO4** |
| | **c.** | Using mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at y(2) taking h = 0.2. Given that y(1) = 2 by Runge-Kutta method of 4th order. | **6** | **L3** | **CO5** |

* * * * *