CBCS SCHEME

USN | | | | | | | | | | |

BMATE101

First Semester B.E./B.Tech. Degree Examination, Jan./Feb. 2023
Mathematics - I for Electrical and Electronics Engineering Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
Q.1	a.	If ϕ is the angle between the radius vector and tangent to the polar curve $r = t(\theta)$, prove that $\tan\phi = r\dfrac{d\theta}{dr}$.	6	L2	CO1
	b.	Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.	7	L2	CO1
	c.	Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$.	7	L2	CO1
		OR			
Q.2	a.	Prove that the polar curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect orthogonally.	6	L2	CO1
	b.	Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \dfrac{\pi}{2}$.	7	L2	CO1
	c.	Using Modern mathematical tool, write a program / code to plot the curve $r = 2\lvert\cos 2\theta\rvert$.	7	L3	CO5
		Module – 2			
Q.3	a.	Expand $f(x) = \cos x + \sin x$ in a Maclaurin series upto the term involving x^5.	6	L2	CO2
	b.	If $U = \log(\tan x + \tan y + \tan z)$, show that $\sin 2x\dfrac{\partial u}{\partial x} + \sin 2y\dfrac{\partial u}{\partial y} + \sin 2z\dfrac{\partial u}{\partial z} = 2$.	7	L2	CO2
	c.	If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.	7	L2	CO2
		OR			
Q.4	a.	Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$.	6	L1	CO2
	b.	If $V = f(x-y, y-z, z-x)$, show that $\dfrac{\partial v}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial v}{\partial z} = 0$.	7	L2	CO2
	c.	Using modern mathematics tool, write a program / code to show that $U_{xx} + U_{yy} = 0$. Given $U_3 = e^x(x\cos y - y\sin y)$.	7	L3	CO2

		Module – 3			
Q.5	**a.**	Solve $(5x^4 + 3x^2y^2 - 2xy^3)\, dx + (2x^3y - 3x^2y^2 - 5y^4)\, dy = 0$.	**6**	**L2**	**CO3**
	b.	Find the Orthogonal trajectory of the family of confocal ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.	**7**	**L3**	**CO3**
	c.	Solve $xyp^2 - (x^2 + y^2)\, p + xy = 0$.	**7**	**L2**	**CO3**
		OR			
Q.6	**a.**	Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$.	**6**	**L2**	**CO3**
	b.	Find the current i at any time t, if initially there is no current in the circuit governed by the differentiate equation $L\left(\frac{di}{dt}\right) + Ri = 200 \operatorname{Sin} 300t$, when $L = 0.05$ and $R = 100$.	**7**	**L3**	**CO3**
	c.	Find the general solution of the equation $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$.	**7**	**L2**	**CO3**
		Module – 4			
Q.7	**a.**	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$, by changing the order of integration.	**6**	**L2**	**CO4**
	b.	Evaluate $\int_{-p}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$.	**7**	**L2**	**CO4**
	c.	Prove that $\beta(m, n) = \frac{\Gamma(m)\,(n)}{\Gamma(m+n)}$.	**7**	**L1**	**CO4**
		OR			
Q.8	**a.**	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)\, dx\, dy$ by changing to polar coordinates.	**6**	**L2**	**CO4**
	b.	Using double integration, find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.	**7**	**L3**	**CO4**
	c.	Show that $\int_0^{\pi/2} \sqrt{\operatorname{Sin}\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\operatorname{Sin}\theta}} = \pi$ by using Beta and Gamma functions.	**7**	**L2**	**CO4**
		Module – 5			
Q.9	**a.**	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$.	**6**	**L2**	**CO5**

	b.	Solve the system of linear equations by Gauss – Jordan method. $x + y + z = 11$ $3x - y + 2z = 12$ $2x + y - z = 3$.	7	L2	CO5
	c.	Determine the largest eigen value and the corresponding eigen vector of the matrix. $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using Power method with initial eigen vector $X_o = [1\ 0\ 0]^T$. Carry out six iterations.	7	L2	CO5
		OR			
Q.10	a.	For what values of λ and μ the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ may have i) Unique solution ii) Infinite number of solution iii) No solution.	6	L2	CO5
	b.	Solve by using Gauss elimination method the equations $3x + y - z = 3$; $2x - 8y + z = -5$; $x - 2y + 9z = 8$.	7	L2	CO5
	c.	Using modern mathematical tool, write a program / code to find the largest eigen value of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by power method.	7	L3	CO5

* * * * *