

# MAKE-UP EXAM

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BMATS101

## First Semester B.E./B.Tech. Degree Examination, Nov./Dec. 2023

### Mathematics – I for CSE Stream

Time: 3 hrs.

Max. Marks: 100

**Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. VTU Formula Hand Book is permitted.

3. M : Marks , L: Bloom's level , C: Course outcomes.

<b>Module – 1</b>			M	L	C
<b>Q.1</b>	<b>a.</b>	With usual notation, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left( \frac{dr}{d\theta} \right)^2$ .	6	L2	CO1
	<b>b.</b>	Find the angle of intersection between the curves $r = ae^\theta$ and $re^\theta = b$ .	7	L2	CO1
	<b>c.</b>	Find the radius of curvature of the curve $x = a \log (\sec t + \tan t)$ , $y = a \sec t$ at any point 't'.	7	L2	CO1

**OR**

<b>Q.2</b>	<b>a.</b>	Show that the curves $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$ cuts each other orthogonally.	6	L2	CO1
	<b>b.</b>	Find the Pedal equation of the curve $r(1 - \cos \theta) = 2a$ .	7	L2	CO1
	<b>c.</b>	Using modern mathematical tool, write a programme / code to plot the sine and cosine curves.	7	L3	CO5

**Module – 2**

<b>Q.3</b>	<b>a.</b>	Expand $\sqrt{1 + \sin 2x}$ upto the term containing $x^4$ using Maclaurin's series.	6	L2	CO2
	<b>b.</b>	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ , then prove that $\frac{1}{2}u_x + \frac{1}{3}u_y + \frac{1}{4}u_z = 0$ .	7	L2	CO2
	<b>c.</b>	Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ .	7	L2	CO2

**OR**

<b>Q.4</b>	<b>a.</b>	Evaluate $\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)^{\frac{1}{x}}$	8	L2	CO1
	<b>b.</b>	If $u = \log \left( \frac{x^2 + y^2}{x + y} \right)$ , show that $xu_x + yu_y = 1$ .	7	L2	CO2
	<b>c.</b>	Using modern mathematical tool, write a programme / code to evaluate $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x$ .	5	L3	CO5

**Module – 3**

<b>Q.5</b>	a.	Solve $\frac{dy}{dx} + y \tan x = y^3 \sec x$ .	6	L2	CO2
	b.	Find the orthogonal trajectories of $r = a(1 - \cos \theta)$ , where $a$ is parameter.	7	L3	CO2
	c.	Find a solution for the non – linear differential equation $xy p^2 - (x^2 + y^2)p + xy = 0$ .	7	L2	CO2

**OR**

<b>Q.6</b>	a.	Solve $(y \cos x + \sin y + y)dx + (\sin x + x \cos y + x) dy = 0$ .	6	L2	CO2
	b.	Find the general solution of the equation $(px - y)(py + x) = 2p$ by reducing into Clairauts form by taking the substitution $X = x^2$ , $Y = y^2$ .	7	L3	CO2
	c.	If the temperature of the air is $30^\circ\text{C}$ and the substance cools from $100^\circ\text{C}$ to $70^\circ\text{C}$ in 5 minutes. Find ‘t’ when the temperature will be $40^\circ\text{C}$ .	7	L2	CO2

**Module – 4**

<b>Q.7</b>	a.	Find the unit digit in the remainder $7^{289}$ .	6	L1	CO3
	b.	Solve the system of linear congruence $x \equiv 2(\text{mod}3)$ , $x \equiv 3(\text{mod}5)$ , $x \equiv 2(\text{mod}7)$ by using CRT.	7	L2	CO3
	c.	Find the remainder when $146!$ is divided by 149.	7	L2	CO3

**OR**

<b>Q.8</b>	a.	Find the remainder when $135 \times 74 \times 48$ is divided by 7.	6	L2	CO3
	b.	Using RSA algorithm decrypt 09810461 using $d = 937$ , $p = 43$ , $q = 59$ .	7	L2	CO3
	c.	Using Fermat's little theorem, find the remainder when $11^{104}$ is divided by 7.	7	L2	CO3

**Module – 5**

<b>Q.9</b>	a.	Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$	6	L2	CO4
	b.	Solve the system of equation by using Gauss – Jordan method. $x + y + z = 9$ , $x - 3y + 4z = 13$ , $3x + 4y + 5z = 40$ .	7	L2	CO4
	c.	Using Power method, find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by considering Initial vector as $[1, 1, 1]^T$ .	7	L2	CO4

## OR

<b>Q.10</b>	<b>a.</b> Solve the following system of equations by Gauss – Seidel method. $x + y + 54z = 110$ , $27x + 6y - z = 85$ , $6x + 15y + 2z = 72$ . Carry out four iterations.	<b>8</b>	<b>L1</b>	<b>CO4</b>
	<b>b.</b> Investigate the values of $\lambda$ & $\mu$ , such that the system of equations $x + y + z = 6$ $x + 2y + 6z = 10$ $x + 2y + \lambda z = \mu$ may have i) Unique solution ii) No solution and iii) Infinitely many solution.	<b>7</b>	<b>L2</b>	<b>CO4</b>
	<b>c.</b> Using modern mathematical tool write a program / code to test the consistency of the equations $x + 2y - z = 1$ ; $2x + y + 4z = 2$ ; $3x + 3y + 4z = 1$ .	<b>5</b>	<b>L3</b>	<b>CO5</b>