CBCS SCHEME

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18MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$. **(06 Marks)**

b. Derive Cauchy Riemann equation in polar form. **(07 Marks)**

c. If f(z) is analytic, prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$. **(07 Marks)**

OR

2 a. Find the analytic function whose imaginary part is $e^x(x\sin y + y\cos y)$. **(06 Marks)**

b. Show that $u = \sin x\cosh y + 2\cos x\sinh y + x^2 - y^2 + 4xy$ is harmonic. Also determine the analytic function f(z). **(07 Marks)**

c. Derive Cauchy Riemann equation in Cartesian form. **(07 Marks)**

Module-2

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Discuss the transformation $\omega = z^2$ **(07 Marks)**

c. Find the bilinear transformation which maps the points $z = \infty, i, 0$ into $\omega = -1, -i, 1$. Also find the fixed points of the transformation. **(07 Marks)**

OR

4 a. Evaluate $\int_C |z|^2 dz$ where C is the square with vertices (0, 0), (1, 0), (1, 1), (0, 1). **(06 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)}$ where C is the circle $|z| = 3$. **(07 Marks)**

c. Find the bilinear transformation which map the points $Z_1 = i$, $Z_2 = 1$, $Z_3 = -1$ onto the points $\omega_1 = 1$, $\omega_2 = 0$, $\omega_3 = \infty$. **(07 Marks)**

Module-3

5 a. The probability distribution of a random variable X is given by the following table:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(i) Find K

(ii) Evaluate $P(X < 6)$ and $P(3 < x \leq 6)$ **(06 Marks)**

b. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that, (i) no line is busy (ii) all lines are busy (iii) at least one line is busy (iv) Atmost 2 lines are busy. **(07 Marks)**

c. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for :
   (i) 10 minutes or more
   (ii) Less than 10 minutes.
   (iii) Between 10 and 12 minutes **(07 Marks)**

OR

6 a. The probability density function of a random variable is,

$$P(x)=\begin{cases} Kx^2, & -3 \le x \le 3 \\ 0, & \text{Otherwise} \end{cases}$$

Find (i) K (ii) $P(1 \le x \le 2)$ (iii) $P(x \le 2)$ **(06 Marks)**

b. The probability that a news reader commits no mistake in reading the news is $\frac{1}{e^3}$. Find the probability that on a particular news broadcast he commits (i) Only 2 mistakes (ii) more than 3 mistakes (iii) atmost 3 mistakes, assuming that mistakes follow Poisson distribution. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be, (i) less than 65, (ii) more than 75 (iii) between 65 and 75. (Given $\phi(1)=0.3413$) **(07 Marks)**

Module-4

7 a. The ranking of 10 students in two subjects, Field theory (A) and Network Analysis (B) are given below:

Roll No. of the students	1	2	3	4	5	6	7	8	9	10
A	3	5	8	4	7	10	2	1	6	9
B	6	4	9	8	1	2	3	10	5	7

Calculate the Rank correlation coefficient. **(06 Marks)**

b. Fit a parabola $y=a+bx+cx^2$ for the data.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

(07 Marks)

c. In a partially destroyed Laboratory record of an analysis. The lines of regression of y on x and x on y are available as $4x-5y+33=0$ and $20x-9y-107=0$. Calculate $\bar{x}, \bar{y}$ and coefficient of correlation between x and y. **(07 Marks)**

OR

8 a. If θ is the angle between the two regression lines, show that

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}$$

(06 Marks)

b. Fit a straight line in the least square sense for the following data:

x	50	70	100	120
y	12	15	21	25

(07 Marks)

c. Find the coefficient of correlation for the data.

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(07 Marks)

Module-5

9 a. Determine (i) Marginal distribution (ii) Covariance between the discrete random variables X and Y along with the joint probability distribution.

X \ Y	1	3	9
2	$1/8$	$1/24$	$1/12$
4	$1/4$	$1/4$	0
6	$1/8$	$1/24$	$1/12$

(06 Marks)

b. In 324 throws of a six faced 'die', an odd number turned up 181 times. Is it possible to think that the 'die' is an unbiased one? **(07 Marks)**

c. A random sample of 10 boys had the following:
I.Q : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Does the data support the assumption of a population mean I.Q of 100 at 5% level of significance?
$\left(\text{Note}: t_{0.05} = 2.262 \text{ for g d.f}\right)$ **(07 Marks)**

OR

10 a. Explain the terms : (i) Null hypothesis (ii) Confidence intervals (iii) Type I and II errors **(06 Marks)**

b. The joint probability of the random variable X and Y as follows :

X \ Y	−4	2	7
1	$1/8$	$1/4$	$1/8$
5	$1/4$	$1/8$	$1/8$

Compute :
(i) E(X) and E(Y)
(ii) E(XY)
(iii) σ_X and σ_Y
(iv) COV(X, Y) **(07 Marks)**

c. Fit a Poisson distribution for the data and test the goodness of fit given that $\chi^2_{0.05} = 7.815$ for 3 d.f

x	0	1	2	3	4
f	122	60	15	2	1

(07 Marks)

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