# Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ 

(06 Marks)

- b. Solve by using Gauss elimination method. Given x + y + z = 9, 2x + y z = 0 and 2x + 5y + 7z = 52. (07 Marks)
- c. Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ . (07 Marks)

OR

- 2 a. Find the rank of the matrix  $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ . (06 Marks)
  - b. Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ . (07 Marks)
  - c. Find the values of  $\lambda$  and  $\mu$  so that the equations x + y + z = 6, x + 2y + 3z = 10 and  $x + 2y + \lambda z = \mu$  have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (07 Marks)

Module-2

- 3 a. Using Newton Raphson method, find the real root of the equation  $3x = \cos x + 1$ , correct to four decimal places. Take x = 0.6 as the initial approximation. (06 Marks)
  - b. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304. Find f(85) using Newton's backward difference interpolation formula. (07 Marks)
  - c. Evaluate  $\int_{0}^{6} \frac{1}{1+x^2} dx$  by using Simpson's  $\frac{1}{3}$  rule by considering 6 subintervals. (07 Marks)

OR

- 4 a. Using Regula Falsi method, find a real root of the equation  $x \log_{10} x 1.2 = 0$  which lies in (2, 3). Carryout 3 iterations. (06 Marks)
  - b. Using the following data, find y when x = 1. Given,

Use Newton's forward interpolation formula.

X	3	4	5	6	7	8	9
у	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(07 Marks)

- c. Evaluate  $\int_{0}^{5.2} \log x \, dx$  by using Weddle's rules taking 6 subintervals.
- (07 Marks)

### Module-3

- 5 a. Solve  $(D^3 + 3D^2 + 3D + 1)y = 0$ . (06 Marks)
  - b. Solve  $(D^2 + 7D + 12)y = \cosh x$ . (07 Marks)
  - c. Solve  $(D^2 4D + 4)y = \cos 2x$ . (07 Marks)

#### OR

- 6 a. Solve  $(4D^4 8D^3 7D^2 + 11D + 6)y = 0$ . (06 Marks)
  - b. Solve  $(D^2 6D + 9)y = 6e^{3x}$ . (07 Marks)
  - c. Solve  $(D^2 5D + 6)y = \sin 3x$ . (07 Marks)

## Module-4

- 7 a. Form the partial differential equation by eliminating arbitrary functions from  $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ . (06 Marks)
  - b. Form the PDE by eliminating arbitrary constants a and b from the relation  $(x-a)^2 + (y-b)^2 + z^2 = k^2$ . (07 Marks)
  - c. Solve  $\frac{\partial^2 z}{\partial x^2} = a^2 z$ , given that when x = 0, z = 0 and  $\frac{\partial z}{\partial x} = a \sin y$ . (07 Marks)

#### OR

- 8 a. Form a partial differential equation by eliminating the arbitrary function from  $\phi(x+y+z, x^2+y^2+z^2)=0$ . (06 Marks)
  - b. Form a partial differential equation by eliminating arbitrary function from z = f(x + ct) + g(x ct). (07 Marks)
  - c. Solve  $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$  by direct integration. Given that  $\frac{\partial z}{\partial y} = -2 \sin y$  when x = 0 and z = 0
    - when y is an odd multiple of  $\frac{\pi}{2}$ . (07 Marks)

## Module-5

- 9 a. Given  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{3}$  and  $P(A \cup B) = \frac{1}{2}$ . Find P(A/B), P(B/A),  $P(A \cap \overline{B})$  and  $P(A/\overline{B})$ .
  - b. The probability that three students A, B, C, solve a problem is  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  respectively. If the problem is simultaneously assigned to all of them, what is the probability that the problem is solved? (07 Marks)
  - c. State and prove Baye's theorem. (07 Marks)

#### OR

- 10 a. If A and B are independent events, show that  $\overline{A}$  and  $\overline{B}$  are also independent. (06 Marks)
  - b. The probability that a team wins a match is  $\frac{3}{5}$ . If this team plays 3 matches in a tournament, what is the probability that the team wins (i) at least one match (ii) all matches. (07 Marks)
  - c. An office has 4 secretaries handling respectively 20%, 60% and 15% and 5% of the files of all government reports. The probability that they misfile such reports is respectively 0.05, 0.1 and 0.05. Find the probability that a misfiled report can be blamed on first secretary?

(07 Marks)