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18MAT31

Third Semester B.E. Degree Examination, June/July 2024
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of
i) $e^{-t}\cos^2 3t$ ii) $t\cos t$ **(06 Marks)**

b. A periodic function of period $\frac{2\pi}{\omega}$ is defined by

$$f(t) = \begin{cases} E\sin\omega t, & 0 \le t \le \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$$ where E and ω are constants.

Show that $L\{f(t)\} = \frac{E\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$ **(07 Marks)**

c. Find the Inverse Laplace transform of
i) $\frac{2s-1}{s^2+2s+17}$ ii) $\log\left(\frac{s^2+1}{s(s+1)}\right)$ **(07 Marks)**

OR

2 a. Express the function f(t) in terms of unit step function and find its Laplace transform, where

$$f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \\ \sin t, & t > 2\pi \end{cases}$$ **(06 Marks)**

b. Using the convolution theorem, obtain inverse Laplace transform of $\frac{s}{(s+1)(s^2+1)}$ **(07 Marks)**

c. Solve the equation $y'' + 5y' + 6y = e^t$ under the condition $y(0) = 0,\ y'(0) = 0$ **(07 Marks)**

Module-2

3 a. Find the Fourier series of the function $f(x) = x^2$ in $(-\pi, \pi)$. **(08 Marks)**

b. Define half range sine and cosine series in the interval $(0, l)$. **(04 Marks)**

c. Find the constant term and the first two harmonics in the fourier series for f(x) given by the following table.

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

OR

4 a. Obtain the fourier series of the saw-tooth function

$f(x) = \frac{Ex}{T}$ for $0 < x < T$ given that $f(x + T) = f(x)$ for all $x > 0$. **(06 Marks)**

b. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} \pi x & \text{in } 0 \le x \le 1 \\ \pi(2-x) & \text{in } 1 \le x \le 2 \end{cases}$$ over the interval (0, 2)

Deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots$ **(07 Marks)**

c. Expand $f(x) = \sin x$ in half range cosine series over the interval $(0, \pi)$. **(07 Marks)**

Module-3

5 a. Prove that fourier transform of

$$f(x) = \begin{cases} 1 + \frac{x}{a}, & -a < x < 0 \\ 1 - \frac{x}{a}, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$ is $\frac{4\sin^2 \frac{au}{2}}{au^2}$, if Fourier transform of f(x) is F(u). **(06 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence

evaluate $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx$, $m > 0$. **(07 Marks)**

c. Find z-transform of $5n^2 + 4\sin\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ **(07 Marks)**

OR

6 a. Find the fourier cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(07 Marks)**

b. Obtain the inverse z-transform of $\frac{4z^2 - 2z}{(z-1)(z-2)^2}$ **(07 Marks)**

c. Solve the difference equation

$u_{n+2} + 3u_{n+1} + 2u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$, using z-transform. **(06 Marks)**

Module-4

7 a. Use Taylor's series method to find the value of y at x = 0.1, given that $dy/dx = x^2 + y^2$, $y(0) = 1$. Consider upto 4^{th} degree term. **(06 Marks)**

b. By using modified Euler's method, solve the initial value problem $\frac{dy}{dx} = \log(x + y)$, $y(1) = 2$ at the point x = 1.2. Take h = 0.2 and carryout two modifications. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$. Find y(0.4) correct to three decimal places using Milne's predictor – corrector method. Apply corrector formula once. **(07 Marks)**

OR

8 a. Using modified Euler's method compute y(1.1) correct to five decimal places taking $h = 0.1$, given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and $y = 1$ at $x = 1$. **(06 Marks)**

b. Use fourth order Runge-Kutta method to find y at $x = 0.1$, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$. **(07 Marks)**

c. Apply Adam's – Bashforth method to solve the equation $(y^2 + 1)dy - x^2\,dx = 0$ at $x = 1$ given $y(0) = 1$, $y(0.25) = 1.0026$, $y(0.5) = 1.0206$, $y(0.75) = 1.0679$. Apply corrector formula once. **(07 Marks)**

Module-5

9 a. By Runge-Kutta method solve $y'' = xy'^2 - y^2$ for $x = 0.2$ correct to four decimal places, using initial conditions $y = 1$ and $y' = 0$ when $x = 0$. Take step length $h = 0.2$. **(06 Marks)**

b. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. Prove that geodesics on a plane are straight line. **(07 Marks)**

OR

10 a. Using Runge-Kutta method solve the differential equation at $x = 0.1$ under the given conditions:

$$\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right), \quad y(0) = 1, \; y'(0) = 0.5.$$ Take step length $h = 0.1$. **(06 Marks)**

b. Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

Apply corrector formula once. **(07 Marks)**

c. Find the extremal of the functional $\int_a^b (x^2y'^2 + 2y^2 + 2xy)dx$ **(07 Marks)**

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