CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, July/August 2022
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform,
(i) $e^{-2t}(2\cos 5t - \sin 5t)$ (ii) $\cosh^2 3t$ **(06 Marks)**

b. Find the Laplace transform of the full wave rectifier $f(t) = E\sin\omega t \quad 0 < t < \frac{\pi}{\omega}$ having a period $\frac{\pi}{\omega}$. **(07 Marks)**

c. Find the inverse Laplace transform $\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$. **(07 Marks)**

OR

2 a. Find the Laplace transform, $\frac{\cos at - \cos bt}{t}$. **(06 Marks)**

b. Solve by using Laplace transform method $y'''(t) + 2y''(t) - y'(t) - 2y(t) = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$ **(07 Marks)**

c. Express the function f(t) in terms of unit step function and hence find its inverse LT,

$$f(t) = \begin{cases} \cos t & 0 < t \le \pi \\ 1 & \pi < t \le 2\pi \\ \sin t & t > 2\pi \end{cases}$$ **(07 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$, in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$. **(06 Marks)**

b. Show that the sine half range series for the function, $f(x) = Lx - x^2$, in $0 < x < L$ is $\frac{8L^2}{\pi^3}\sum_0^\infty \frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{L}\right)\pi x$. **(07 Marks)**

c. Obtain the Fourier series of y up to the first harmonics for the following values :

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	−1.5	0	2.6	3.4

(07 Marks)

OR

4 a. Expand the function $f(x) = x\sin x$, as a Fourier series in the interval $-\pi \le x \le \pi$. Deduce that $\frac{1}{1,3} - \frac{1}{3,5} + \frac{1}{5,7} ...... = \frac{\pi - 2}{4}$ **(06 Marks)**

b. Obtain the half range cosine series of $f(x) = x\sin x$ $0 \le x \le \pi$. **(07 Marks)**

c. Obtain the constant term and the first three coefficients in the Fourier cosine series for y using the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

5 a. Find the complex Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$.

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. If $\overline{f(z)} = \frac{2z^2 + 3z + 12}{(z-1)^4}$ find the value of u_0, u_1, u_2, u_3 **(07 Marks)**

c. Solve by using z-transforms, $u_{n+2} + 5u_{n+1} + 6u_n = 2^n$: $u_1 = 0$, $u_0 = 0$ **(07 Marks)**

OR

6 a. Find the Fourier sine transform of e^{-ax}, $a > 0$. **(06 Marks)**

b. Find the Fourier sine and cosine transform of $2e^{-3x} + 3e^{-2x}$. **(07 Marks)**

c. Solve by using Z-transforms,
$y_{n+2} + 2y_{n+1} + y_n = n$, with $y(0) = 0 = y$ **(07 Marks)**

Module-4

7 a. Use Taylor's series method to find y(4.1) given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$ and $y(4) = 4$. **(06 Marks)**

b. Use Fourth order Runge-Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, $y(0.4) = 1$ at $x = 0.5$. Correct to four decimal places. **(07 Marks)**

c. The following table gives the solution of $5xy^1 + y^2 - 2 = 0$, find the value of y at x = 4.5 using Milne's Predictor and Corrector formulae, use the corrector formulae twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

(07 Marks)

OR

8 a. Using modified Euler's method find y at x = 0.2 given $\frac{dy}{dx} = 3x + \frac{y}{2}$, with $y(0) = 1$ taking $h = 0.1$. **(06 Marks)**

b. Using Runge-Kutta method of fourth order find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, y(0) = 1 taking h = 0.2 **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $(y^2 + 1)dy - x^2dx = 0$, at x = 1, given y(0) = 1, y(0.25) = 1.0026, y(0.5) = 1.0206, y(0.75) = 1.0679. Apply the corrector formulae twice. **(07 Marks)**

Module-5

9 a. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$, Evaluate $y(0.1)$ using Runge-Kutta method of order 4. **(06 Marks)**

b. A necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be extremum that $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. Show that the extremal of the functional $\int_0^1 y^2\{3x(y'^2 - 1) + yy'^3\}dx$, subject to the conditions $y(0) = 0$, $y(1) = 2$, is the circle $x^2 + y^2 - 5x = 0$. **(07 Marks)**

OR

10 a. Apply Milne's method to compute $y(0.8)$. Given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values. **(06 Marks)**

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

b. Find the extremal of the functional $\int_a^b (x^2y'^2 + 2y^2 + 2xy)dx$. **(07 Marks)**

c. Prove that Geodesics on a plane are straight line. **(07 Marks)**

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