CBCS SCHEME

USN | | | | | | | | | |

18MATDIP31

Third Semester B.E. Degree Examination, June/July 2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{n\theta}{2}\right)$ **(07 Marks)**

b. Express $\sqrt{3}+i$ in the polar form and hence find its modulus and amplitude. **(07 Marks)**

c. Find the argument of $\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ **(06 Marks)**

OR

2 a. If $\vec{A}=i+2j+3k$, $\vec{B}=-i+2j+k$ and $\vec{C}=3i+j$, find P such that $\vec{A}+P\vec{B}$ is perpendicular to $\vec{C}$. **(07 Marks)**

b. Find the area of the parallelogram whose adjacent sides are the vectors $\vec{A}=2i+4j-5k$ and $\vec{B}=i+2j+3k$. **(06 Marks)**

c. If $\vec{A}=4i+3j+k$ and $\vec{B}=2i-j+2k$, find a unit vector N form a right handed system. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's series expansion of $\sin x$ up to term containing x^4. **(07 Marks)**

b. If $U=\sin^{-1}\left[\frac{x^2+y^2}{x-y}\right]$, prove that $x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=\tan U$. **(07 Marks)**

c. If $U=f(x-y,\ y-z,\ z-x)$ prove that $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}=0$. **(06 Marks)**

OR

4 a. Prove that $\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+........$ by using Maclaurin's series notation. **(07 Marks)**

b. Using Euler's theorem prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=4u\log u,\ \text{if } u=e^{\left(\frac{x^3y^3}{x^2+y^2}\right)}$$ **(07 Marks)**

c. If $u=x+y$, $v=y+z$ and $w=z+x$ then find $J\left(\frac{u,v,w}{x,y,z}\right)$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$, where t is the time variable. Determine the components of velocity and acceleration vectors at $t = 0$ in the direction of $i + j + k$. **(07 Marks)**

b. Find the unit normal to the surface $x^2y + 2xz = 4$ at $(2, -2, 3)$. **(06 Marks)**

c. Show that the vector field $\vec{F} = (4xy - z^3)i + (2x^2)j - (3xz^2)k$ is irrotational. **(07 Marks)**

OR

6 a. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

b. If $\vec{F} = (3x^2y - z)i + (xz^3 + y^4)j - 2x^3z^2k$, find $\text{grad}(\text{div}\vec{F})$ at $(2, -1, 0)$. **(07 Marks)**

c. Find the value 'a' such that the vector field $\vec{F} = (x + 3y)i + (y - 2z)j + (x + az)k$ is Solenoidal. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, $n > 0$. **(07 Marks)**

b. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}\,dx$ **(06 Marks)**

c. Evaluate $\iint_C xy(x + y)\,dxdy$ over the area between $y = x^2$ and $y = x$. **(07 Marks)**

OR

8 a. Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x\,dx, \quad n > 0$$ **(07 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}}\,dx$ **(06 Marks)**

c. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ **(07 Marks)**

Module-5

9 a. Solve $(4xy + 3y^2 - x)\,dx + x(x + 2y)\,dy = 0$ **(07 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$ **(06 Marks)**

c. Obtain the solution of the differential equation

$$(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$ **(07 Marks)**

OR

10 a. Solve : $\tan y\,dy = (\cos y \cos^2 x - \tan x)dx$ **(07 Marks)**

b. Solve : $\left[y\left(1 + \frac{1}{x}\right) + \cos y\right]dx + (x + \log x - x\sin y)dy = 0$ **(07 Marks)**

c. Solve : $(1 + y^2)dx = (\tan^{-1}y - x)\,dy$ **(06 Marks)**

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