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18MATDIP31

Third Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express $\dfrac{(3+i)(1-3i)}{(2+i)}$ in the form x + iy. **(06 Marks)**

b. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$. Find the value of 'ρ' such that $\vec{a} - \rho\vec{b}$ is perpendicular to $\vec{c}$. **(07 Marks)**

c. Find the angle between the vector $\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **(07 Marks)**

OR

2 a. Find the modulus and amplitude of the complex number $1 + \cos\alpha + i\sin\alpha$. **(06 Marks)**

b. Prove that $\left(\dfrac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n = \cos n\theta + i\sin n\theta$. **(07 Marks)**

c. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

Module-2

3 a. Find the n^{th} derivative of cosx cos2x. **(06 Marks)**

b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ upto the term containing x^4. **(07 Marks)**

c. If $u = f(y-z, z-x, x-y)$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. **(06 Marks)**

b. If $z = xy^2 + x^2y$ where $x = at^2$ and $y = 2at$. Find $\dfrac{dz}{dt}$. **(07 Marks)**

c. If $x = e^u \sec v$, $y = e^u \tan v$. Find $J\left(\dfrac{x, y}{u, v}\right)$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve
$\vec{r} = \cos 2t\hat{i} + \sin 2t\hat{j} + t\hat{k}$ where t is the time variable. Determine the components of velocity and acceleration vectors at $t = \pi/8$ in the direction of $\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \hat{k}$. **(06 Marks)**

b. Find $\text{div}\vec{f}$ for $\vec{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Show that $\vec{f} = (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2xz)\hat{k}$ is irrotional and find φ such that $\vec{f} = \nabla\varphi$. **(07 Marks)**

OR

6 a. Find the unit normal to the surface $x^3y^3z^2 = 4$ at the point P(-1, -1, 2). **(06 Marks)**

b. If $\vec{f} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\varphi = 2z - x^3y$, find $\vec{f} \bullet (\nabla\varphi)$ and $\vec{f} \times (\nabla\varphi)$ at (1, -1, 1). **(07 Marks)**

c. Show that $\vec{f} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

Module-4

7 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x \, dx \ (n > 0)$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$. **(07 Marks)**

OR

8 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x \, dx \ (n > 0)$. **(06 Marks)**

b. Evaluate $\iint_R xy \, dx \, dy$ where R is the first quadrant of the circle $x^2 + y^2 = a^2$, $x \geq 0$, $y \geq 0$. **(07 Marks)**

c. Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z)\, dy \, dx \, dz$. **(07 Marks)**

Module-5

9 a. Solve $x^2 \dfrac{dy}{dx} - 2xy - x + 1 = 0$. **(06 Marks)**

b. Solve $(3x^2y^2 + x^2)dx + (2x^3y + y^2)dy = 0$. **(07 Marks)**

c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. **(07 Marks)**

OR

10 a. Solve $\left[y\left(1 + \dfrac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]dy = 0$. **(06 Marks)**

b. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. **(07 Marks)**

c. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2x$. **(07 Marks)**

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