CBCS SCHEME

USN | | | | | | | | | | |

18MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express the complex number $\frac{5+5i}{3-4i}$ in the form x + iy. **(06 Marks)**

b. Find the amplitude and modulus of the complex number $\frac{4+2i}{2-3i}$ **(07 Marks)**

c. Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}.\cos^n\frac{\theta}{2}\cos\left(\frac{n\theta}{2}\right)$ **(07 Marks)**

OR

2 a. Show that the points A(–1, 4, –3), B(3, 2, –5), C(–3, 8, –5) and D(–3, 2, 1) are coplanar. **(06 Marks)**

b. Find the cube roots of $1 - i$ **(07 Marks)**

c. Find the sine of the angle between $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. **(07 Marks)**

Module-2

3 a. Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\ldots\ldots$ by using Maclaurin's series. **(06 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} + z.\frac{\partial u}{\partial z} = 0$ **(07 Marks)**

c. If $u = \tan^{-1}\left(\frac{x^3y^3}{x^3+y^3}\right)$ prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = \frac{3}{2}\sin 2u$. **(07 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of e^x. **(06 Marks)**

b. If $u = e^{x^3+y^3}$ prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = 3u\log u$ **(07 Marks)**

c. If $u = x - y$ and $v = \frac{1}{x-y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$ **(07 Marks)**

Module-3

5 a. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

b. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time. Determine the component of velocity and acceleration at t = 0 in the direction of $\hat{i} + \hat{j} + \hat{k}$. **(07 Marks)**

c. Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at t = 2 and t = 3. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Find div$\vec{F}$ and curl$\vec{F}$ where $\vec{F} = \nabla(xy + yz + zx)$ **(06 Marks)**

b. Find the constants a, b, c such that the vector $\vec{F} = (x + y + az)\hat{i} + (x + cy + 2z)\hat{k} + (bx + 2y - z)\hat{j}$ is irrotational. **(07 Marks)**

c. If $\vec{F} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\phi = 2z - x^3y$, find $\vec{F}\cdot(\nabla\phi)$ and $\vec{F}\times(\nabla\phi)$ at $(1, -1, 1)$. **(07 Marks)**

Module-4

7 a. Find the reduction formula for $\int \cos^n x.dx \;:\; n > 0$ **(06 Marks)**

b. Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}}\,dx$ **(07 Marks)**

c. Evaluate $\int_0^1 \int_{x^2}^{x} (x^2 + 3y + 2)\,dy.dx$ **(07 Marks)**

OR

8 a. Find the reduction formula for $\int \sin^n x.dx \;:\; n > 0$ **(06 Marks)**

b. Evaluate $\int_0^{\pi} x.\cos^6 x.dx$ **(07 Marks)**

c. Evaluate $\int_0^a \int_0^a \int_0^a e^{x+y+z}\,dx\,dy\,dz$ **(07 Marks)**

Module-5

9 a. Solve : $\frac{dy}{dx} = -\frac{y}{x} + y^2x$ **(06 Marks)**

b. Solve $y \sin 2x\,dx - (1 + y + \cos^2 x)dy = 0$ **(07 Marks)**

c. Solve $x.\frac{dy}{dx} + y = x^3y^6$ **(07 Marks)**

OR

10 a. Solve $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - \sin y}$ **(06 Marks)**

b. Solve $dx + x\,dy = e^{-y}.\sec y\,dy$ **(07 Marks)**

c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$ **(07 Marks)**

* * * * *