CBCS SCHEME

USN

18MAT11

# First Semester B.E. Degree Examination, June/July 2023
## Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***

### Module-1

1 a. With usual notations, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(06 Marks)**

b. Find the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ at the point (a, 0). **(06 Marks)**

c. For the curve $\theta = \frac{1}{a}\sqrt{r^2 - a^2} - \cos^{-1}\left(\frac{a}{r}\right)$, prove that $p^2 = r^2 - a^2$. **(08 Marks)**

**OR**

2 a. Find the angle between the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$. **(06 Marks)**

b. Find the radius of curvature of the curve $r = a\sin n\theta$ at the pole (0, 0). **(06 Marks)**

c. Find evolutes curve $y^2 = 4ax$ as $27ay^2 = 4(x+a)^3$. **(08 Marks)**

### Module-2

3 a. Obtain Maclaurin's expansion of $e^{\tan^{-1}x}$ upto the term containing $x^4$. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$. **(06 Marks)**

c. Find the extreme values of $f(x, y) = x^3y^2(1-x-y)$. **(08 Marks)**

**OR**

4 a. If U = f(x-y, y-z, z-x), prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If $u = x + 3y^2 - z^2$, $v = x^2yz$, $w = 2z^2 - xy$, find the Jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at (1, -1, 0). **(06 Marks)**

c. Find the stationary values of $x^2 + y^2 + z^2$ subject to the condition $xy + yz + zx = 3a^2$. **(08 Marks)**

### Module-3

5 a. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\,dx\,dy$. **(07 Marks)**

b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, above x-axis. **(07 Marks)**

c. With usual notations, prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**OR**

6 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} \, dx \, dy$ by changing the order of integration. **(07 Marks)**

b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ by changing into polar coordinates. **(07 Marks)**

c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta} \, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(06 Marks)**

**Module-4**

7 a. Solve $p(p+y) = x(x+y)$. **(07 Marks)**

b. Find the orthogonal trajectories to the family of curve, $y^2 = 4ax$. **(07 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(06 Marks)**

**OR**

8 a. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(07 Marks)**

b. A body originally at $80^\circ$C cools down to $60^\circ$C in 20 minutes, the temperature of the air being $40^\circ$C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

c. Find the general solution and singular solution of the equation $\sin px \cos y = \cos px \sin y + p$. **(06 Marks)**

**Module-5**

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to row-reduced echelon form. **(06 Marks)**

b. Apply Gauss-elimination method to solve the $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$. **(07 Marks)**

c. Find numerically largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. Take initial eigen vector $[1,0,0]^T$. Carry out five iterations. **(07 Marks)**

**OR**

10 a. Test for consistency and solve the system of equations, $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$. **(06 Marks)**

b. Solve the system of equations by Gauss-Seidel method $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. Carryout three iterations. **(07 Marks)**

c. Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(07 Marks)**

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