CBCS SCHEME

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18MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2023
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the angle between the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$. **(06 Marks)**

b. Prove that the pedal equation to the curve $r^m = a^m \cos m\theta$ is $pa^m = r^{m+1}$. **(07 Marks)**

c. Shat that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(07 Marks)**

OR

2 a. Find the pedal equation to the cardioid $r = a(1 + \cos\theta)$. **(06 Marks)**

b. With usual notations prove that $\tan\phi = r\left(\dfrac{d\theta}{dr}\right)$. **(07 Marks)**

c. Find the radius of curvature of the curve $y^2 = \dfrac{a^2(a-x)}{x}$, where the curve meets X – axis. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} \ldots\ldots$ **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Expand $\log(1 + \cos x)$ by Maclaurin's series upto term containing x^4. **(06 Marks)**

b. Find the extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(07 Marks)**

c. If $u = x + y + z$, $v = y + z$, $uvw = z$, find the value of $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dz\,dy\,dx$. **(06 Marks)**

b. Evaluate $\int_{0}^{1}\int_{x}^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Prove that $\beta(m, n) = \dfrac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. **(06 Marks)**

b. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **(07 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. **(07 Marks)**

Module-4

7 a. Solve $[\cos x \tan y + \cos(x + y)]dx + [\sin x \sec^2 y + \cos(x + y)]dy$. **(06 Marks)**

b. Solve $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 y}{y^2}$. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes. If the temperature of the air is 40°C, find the temperature of the body after 40 minutes from the original. **(07 Marks)**

OR

8 a. Solve $y(2x - y + 1)\,dx + x(3x - 4y + 3)\,dy = 0$. **(06 Marks)**

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self Orthogonal. **(07 Marks)**

c. Solve $p(p + y) = x(x + y)$. **(07 Marks)**

Module-5

9 a. Find the rank of $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by Elementary row transformation. **(06 Marks)**

b. Apply Gauss – Jordan method to solve the system of equations.

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$x + y + z = 9$. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix.

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Power method, taking the initial eigen vector as $[1, 1, 1]^{I}$. Perform 5 iterations. **(07 Marks)**

OR

10 a. Solve the following system of equations by Gauss Elimination method.

$2x + y + 4z = 12$

$4x + 11y - z = 33$

$8x - 3y + 2z = 20$. **(06 Marks)**

b. Solve the following system of equations by Gauss Seidel method.

$10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$. **(07 Marks)**

c. Diagonalise the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

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