CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, June/July 2024
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the angle between surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, -1, 2). **(06 Marks)**

b. Evaluate div $\vec{F}$ and curl $\vec{F}$ for the vector point function $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ **(07 Marks)**

c. Determine the constants a, b and c so that the vector,

$$\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$$

is irrotational and find ϕ such that $\vec{F}=\nabla\phi$ **(07 Marks)**

OR

2 a. If $\vec{F}=(3x^2+6y)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$, evaluate $\int \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x=t,\ y=t^2,\ z=t^3$. **(06 Marks)**

b. Using Green's theorem, evaluate $\int_C [(y-\sin x)dx+\cos x dy]$, where c is the plane triangle enclosed by the lines $y=0,\ x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$ **(07 Marks)**

c. Evaluate $\int_s \vec{f}\cdot\hat{n}ds$ where $\vec{f}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and s is the surface of the cube bounded by $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$, by using the Gauss divergence theorem. **(07 Marks)**

Module-2

3 a. Solve : $4\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}-23\frac{d^2y}{dx^2}+12\frac{dy}{dx}+36y=0$. **(06 Marks)**

b. Find the solution of $(D^2-4D+4)y=8(e^{2x}+\sin 2x)$ by inverse operator method. **(07 Marks)**

c. Obtain the solution for the differential equation, $y''-2y'+y=\frac{e^x}{x}$ by the method of variation of parameter. **(07 Marks)**

OR

4 a. Find the solution for the differential equation $\frac{d^3y}{dx^3}+4\frac{dy}{dx}=\sin 2x$. **(06 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+9y=3x^2+\sin(3\log x)$ **(07 Marks)**

c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2}+\omega_0^2 x = F_0 \sin t$, where ω_0 and F_0 are constants. Solve it when $x = 0$, $\frac{dx}{dt} = 0$ initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function f from $lx + my + nz = f(x^2 + y^2 + z^2)$ **(06 Marks)**

b. Solve by direct integration method.
$$\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$$ **(07 Marks)**

c. Derive one dimensional heat equation in the form $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function and from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$ **(06 Marks)**

b. Find the solution of the partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ **(07 Marks)**

c. Obtain all possible solutions of one dimensional wave $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$ by the method of variable separable method. **(07 Marks)**

Module-4

7 a. Test the convergence of the series,
$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + ...\infty$$ **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation which leads to $J_n(x)$. **(07 Marks)**

c. Express the polynomial $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

OR

8 a. Test for convergence for, $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$. **(06 Marks)**

b. Prove that $\int_0^1 xJ_n(\alpha x).J_n(\beta x)dx = 0$ for $\alpha \neq \beta$ and α, β are the roots of the equation $J_n(x) = 0$. **(07 Marks)**

c. Using the Rodrigue's formula, find the Legendre polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. **(07 Marks)**

Module-5

9 a. Compute the real root of $x\log_{10} x - 1.2 = 0$ by the method of false position. Correct to 3 decimal places, which lies between 2 and 3. **(06 Marks)**

b. Using the Newton's divided difference method find the interpolating polynomial of the given data :

x	-1	0	1	3
f(x)	2	1	0	-1

(07 Marks)

c. By using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^6 \frac{dx}{1+x^2}$ by considering seven ordinates. (07 Marks)

OR

10 a. Use Newton-Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \pi$ correct to three decimal places. (06 Marks)

b. From the following table, estimate the number of students who obtained marks between 40 and 45. (07 Marks)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

c. Evaluate $\int_4^{5.2} \log x \, dx$, by using the Weddle's Rule taking Seven ordinates. (07 Marks)

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