Second Semester B.E. Degree Examination, Feb./Mar.2022 **Advanced Calculus and Numerical Methods**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

- a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 3$ at the point 1 (06 Marks)
 - Find the divergence and curl of the vector \overrightarrow{F} if $\overrightarrow{F} = \nabla(x^3 + y^3 + z^3 3xyz)$. (07 Marks)
 - Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational and also find a scalar function ϕ such that $\vec{F} = \nabla \phi$ (07 Marks)

- Verify Green's theorem for $\int (xy + y^2)dx + x^2dy$, where C is the bounded by y = x and (06 Marks)
 - b. Using Stoke's theorem, evaluate $\int xydx + xy^2dy$, where C is the square in the x-y plane with vertices (1, 0)(-1, 0)(0, 1)(0, -1). (07 Marks)
 - c. Using Gauss divergence theorem, evaluate $\iint F \, \overline{n} \, ds$ over the entire surface of the region above xy-plane bounded by the cone $z^2 = x^2 + y^2$ and the plane z = 4, where $\vec{F} = 4xz \vec{i} + xyz^2 \vec{j} + 3z \vec{k}$. (07 Marks)

Module-2

Solve $(D^2 - 4D + 13)y = \cos 2x$, where $D = \frac{d}{dx}$.

(06 Marks)

b. Solve $(D^2 - 2D + 1)y = \frac{e^x}{x}$, by the method of variation of parameter, where $D = \frac{d}{dx}$

(07 Marks)

c. Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$.

(07 Marks)

a. Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x)$, where $D = \frac{d}{dx}$.

(06 Marks)

b. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+x)].$

(07 Marks)

c. The differential equation of the displacement x(t) of a spring fixed at the upper end and a weight at its lower end is given by $10\frac{d^2x}{dt^2} + \frac{dx}{dt} + 200x = 0$. The weight is pulled down 0.25 cm, below the equilibrium position and then released. Find the expression for the displacement of the weight from its equilibrium position at any time t during its first upward motion. (07 Marks)

Module-3

- 5 a. Form the partial differential equation by eliminating the arbitrary constants form, $(x-a)^2 + (y-b)^2 + z^2 = C^2$. (06 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when x = 0 and z = 0 if y is an odd

multiple of $\frac{\pi}{2}$. (07 Marks)

c. Derive one-dimensional heat equation in the standard form. (07 Marks)

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- 6 a. Form the partial differential equation by eliminating the arbitrary function from z = f(x+ct) + g(x-ct) (06 Marks)
 - b. Solve (y-z)p + (z-x)q = (x-y). (07 Marks)
 - c. Solve one dimensional wave equation, using the method of separation of variables.

(07 Marks)

Module-4

- 7 a. Test for the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$. (06 Marks)
 - b. Solve Bessel's differential equation leading to $J_n(x)$. (07 Marks)
 - c. Express $x^4 2x^3 + 3x^2 4x + 5$ in terms of legendre polynomial. (07 Marks)

OR

- 8 a. Discuss the nature of the series, $\frac{1}{2} + \left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots$ (06 Marks)
 - b. With usual notation, show that
 - (i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (07 Marks)

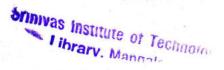
c. Use Rodrigues formula to show that $P_4(\cos\theta) = \frac{1}{64} [35\cos 4\theta + 20\cos 2\theta + 9]$. (07 Marks)

Module-5

- 9 a. Find a real root of the equation $\cos x 3x + 1 = 0$, correct to 3 decimal places using regula falsi method. (06 Marks)
 - b. Use an appropriate interpolation formula to compute f(42) using the following data:

X	40	50	×60	70	80	90
f(x)	184	204	226	250	276	304

(07 Marks)



c. Evaluate \(\log x dx \) by using Weddle's rule, divided into six equal parts.

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(07 Marks)

OR

Find a real root of the equation, $x \sin x + \cos x = 0$ near $x = \pi$, correct to four decimal 10 places. Using Newton-Raphson method. (06 Marks)

b. Find f(9) from the day by Newton's divided difference formula.

(07 Marks)

13 11 f(x)150 392 1452 2366 5202

c. By using Simpson's $\frac{1}{3}^{rd}$ rule \int_{0}^{1} dividing interval (0, 1) into six equal parts and hence

find approximate value of π

(07 Marks)