CBCS SCHEME

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20MCM11

First Semester M.Tech. Degree Examination, July/August 2021
Numerical Methods for Engineers

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to four significant digits and find its absolute and relative errors. **(07 Marks)**

b. By using Bisection method, obtain an approximate root of the equation $x^3 - 2x - 5 = 0$. Carrying out six iterations. **(06 Marks)**

c. Given that the equation $x^{2.2} = 69$ has a root between 5 and 8 use the method of Regula –Falsi to determine it. **(07 Marks)**

2 a. Using Newton – Raphson method, obtain an approximate root of the equation $\log_{10} x = 1.2$. Take $x_0 = 2$. **(06 Marks)**

b. Using secant method, obtain an approximate root of the equation $x_e^x = 1$. **(07 Marks)**

c. Find a positive root of the equation $2x = \cos x + 3$ using fixed point iteration method correct to 3 decimal places. **(07 Marks)**

3 a. Perform three iterations of Muller's method to find the smallest positive root of the equation $f(x) = x^3 - 13x - 12$ with $x_0 = 4.5$, $x_1 = 5.5$ and $x_2 = 5$. **(10 Marks)**

b. Calculate first and second derivatives of the function tabulated in the following table at the pint $x = 1.2$.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(10 Marks)

4 a. Evaluate : $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method correct to four decimal places with h = 0.5, h = 0.25 and h = 0.125. **(10 Marks)**

b. Evaluate $\int_4^{5.2} \log_e x \, dx$ by taking seven ordinates using :

i) Simpson's $\frac{1}{3}^{rd}$ rule

ii) Simpson's $\frac{3}{8}^{th}$ rule

iii) Weddle's rule. **(10 Marks)**

5 a. Solve the given system of equations using Cramer's rule :

$$2x_2 + 5x_3 = 9$$
$$2x_1 + x_2 + x_3 = 9.$$
$$3x_1 + x_2 = 10$$

(05 Marks)

b. Apply Cholesky method, to solve the system of equations :

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6.$$
$$3x + y + 2z = 8$$

(07 Marks)

c. Solve the system of equations :

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

By the Gauss-Jordan method. (08 Marks)

6 a. Determine the inverse of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}.$$

By using the Partition method. (10 Marks)

b. Solve :

$$10x_1 + 2x_2 - x_3 = 27$$
$$-3x_1 - 6x_2 + 2x_3 = -61.5.$$
$$x_1 + x_2 + 5x_3 = -21.5$$

By Gauss elimination method. (10 Marks)

7 a. Using Jacobi's method, find all the eigen values and the corresponding eigen vector of the matrix :

$$A = \begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & \sqrt{2} \end{bmatrix}.$$

(08 Marks)

b. Using Given's method, transform the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to tridiagonal form. (05 Marks)

c. Find the dominant eigen value and the corresponding eigen vector of the matrix :

$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ using power method, taking the initial eigen vector as $[1 \ 0 \ 0]^T$. (07 Marks)

8 a. Find all the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ using the Rutishauser method. **(10 Marks)**

b. Using Householder's method, reduce the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$ into a tridiagonal matrix. **(10 Marks)**

9 a. Give the properties of linear transformation. **(04 Marks)**

b. If $T : R^n \to R^m$ be a linear transformation prove that :
i) T is one – to – one if and only if the equation $T(X) = 0$ has trivial solution.
ii) T maps R^n on to R^m if and only if the columns of A span R^m
iii) T is one – to – one if and only if the columns of A are linearly independent. **(08 Marks)**

c. Find a least square solution to the inconsistent system $AX = b$. where,

$$A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$ **(08 Marks)**

10 a. Give a geometrical interpretation of orthogonal projection.
If $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ find the orthogonal projection of y on to u, then write y as the sum of two orthogonal vectors one is span of (u) and one is orthogonal to u. **(10 Marks)**

b. Discuss in brief Gram – Schmidt process. Show that $\{v_1, v_2, v_3\}$ is an orthonormal basis of

$$R_3 \text{ where } v_1 = \begin{bmatrix} \frac{3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{bmatrix} v_2 = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} \frac{-1}{\sqrt{66}} \\ \frac{-4}{\sqrt{66}} \\ \frac{7}{\sqrt{66}} \end{bmatrix}.$$ **(10 Marks)**

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