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20SCS/SCN/SCE/SSE/SIT/SIS/SFC/SAM11

First Semester M.Tech. Degree Examination, Jan./Feb. 2023
Mathematical Foundations of Computer Science

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Define vector space and subspace. **(06 Marks)**

b. Define linear transformation. Find the linear transformation of $f: R^2 \to R^2$. Such that $f(1,\ 1) = (0,\ 1)$ and $f(-1,\ 1) = (3,\ 2)$ **(07 Marks)**

c. Show that the vectors $(1,\ 1,\ 2,\ 4)$, $(2,\ -1,\ -5,\ 2)$, $(1,\ -1,\ -4,\ 0)$ and $(2,\ 1,\ 1,\ 6)$ are linearly dependent in R^4. **(07 Marks)**

OR

2 a. Define linearly independent and linearly dependent vectors. **(06 Marks)**

b. Define the terms basis and dimension. Find the dimension and basis of the spanned by the vectors $(2,\ 4,\ 2)$, $(1,\ -1,\ 0)$, $(1,\ 2,\ 1)$ and $(0,\ 3,\ 1)$ in $V_3(R)$. **(07 Marks)**

c. Let U and W are the subspaced R^4 generated for set containing
$U = \{(1,\ 1,\ 0,\ -1), (1,\ 2,\ 3,\ 0), (2,\ 3,\ 3,\ -1)\}$ and
$W = \{(1,\ 2,\ 2,\ -2), (2,\ 3,\ 2,\ -3), (1,\ 3,\ 4,\ -3)\}$
Find $\dim(U+W)$ and $\dim(U \cap W)$. **(07 Marks)**

Module-2

3 a. Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$. **(06 Marks)**

b. Find an orthogonal basis for the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ by Gram-Schmidt process. **(07 Marks)**

c. Find a QR factorization of $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$. **(07 Marks)**

OR

4 a. Let $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\{u_1,\ u_2\}$ is an orthogonal basis for $W = \text{span}\{u_1, u_2\}$. Write Y as the sum of a vector in 'W' and a vector orthogonal to 'W'. **(06 Marks)**

b. Find a least square solution of the system of equation AX=b, where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. **(07 Marks)**

c. Find a QR-factorization of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. **(07 Marks)**

Module-3

5 a. Orthogonally diagonalize the matrix $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & -2 & -2 \\ 4 & -2 & 1 \end{bmatrix}$. **(10 Marks)**

b. Find the maximum and minimum value of $Q(X) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $X^TX = 1$. **(10 Marks)**

OR

6 a. Given the following data use the principal component analysis to reduce the dimensions from 2 to 1.

Features	Example 1	Example 2	Example 3	Example 4
x	4	8	13	7
y	11	4	5	14

(10 Marks)

b. Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. **(10 Marks)**

Module-4

7 a. Compute the coefficient of correlation between x and y using the following data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

b. $8x - 10y + 66 = 0$ and $40x - 18y = 214$ are the two regression lines. Find the mean of x's, y's and the correlation co-efficient. Find σ_y if $\sigma_x = 3$. **(07 Marks)**

c. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at x = 6.

x	1	2	3	4	5
y	10	12	13	16	19

(07 Marks)

OR

8 a. Given

	x-series	y-series
Mean	18	100
SD	14	20

and r = 0.8

Write the equation of lines of regression and hence find the most probable value of 'y' when x = 70. **(06 Marks)**

b. Show that if 'θ' is the angle between the lines of regression, then $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. **(07 Marks)**

c. An experiment on lifetime 't' of cutting tool as different cutting speeds V (units) are given below :

Speed (V)	350	400	500	600
Life (t)	61	26	7	2.6

Fit a relation of the form $V = at^b$ **(07 Marks)**

Module-5

9 a. A random variable X has the following probability function for various values of 'x'.

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(i) Find K
(ii) Evaluate $P(x \geq 6)$
(iii) $P(3 < x \leq 6)$ **(06 Marks)**

b. A random variable X take the values $-3, -2, -1, 0, 1, 2, 3$ such that $P(X=0) = P(X<0)$ and $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$. Find the probability distribution. **(07 Marks)**

c. 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit $(\psi^2_{0.05} = 9.49 \text{ for } 4 \text{ d.f})$.

Number of heads	0	1	2	3	4
Frequency	5	29	36	25	5

(07 Marks)

OR

10 a. The probability density function of a variable 'X' is,

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K and $P(X \geq 5)$, $P(3 < X \leq 6)$. **(06 Marks)**

b. A group of boys and girls were given an intelligence test. The mean score, S.D. score and numbers in each group are as follows:

	Boys	Girls
Mean	74	70
S.D	8	10
n	12	10

Is the difference between the means of the two groups significant at 5% level of significance $(t_{0.5} = 2.086 \text{ for } 20 \text{ d.f})$. **(07 Marks)**

c. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories $(\psi^2_{0.05} = 7.81 \text{ for } 3 \text{ d.f})$ **(07 Marks)**

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