## GBGS SCHEME

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## 20SCN/SCE/SSE/SCS/SIT/SIS/SFC/SAM/SAD/SDS11

## First Semester M.Tech. Degree Examination, Feb./Mar. 2022 Mathematical Foundations of Computer Science

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define subspace.

(03 Marks)

- b. Prove that if  $w_1$  and  $w_2$  are subspaces of vector space V(F) then  $w_1 + w_2$  is a subspace of V(F).
- c. Show that the set  $S = \{t^2 + 1, t 1, 2t + 2\}$  is a basis for the vector space  $P_2$ .

(10 Marks)

OR

2 a. Define linear span.

(03 Marks)

- b. Let  $S = \{(1,-3,2), (2,4,1), (1,1,1)\}$  be a subset of  $V_3(R)$ . Show that the vectors (3,-7,6) is in L[S].
- C. Let  $S = \{V_1, V_2, V_3, V_4\}$  be a basis for  $R^4$ , where  $V_1 = (1, 1, 0, 0), V_2 = (2, 0, 1, 0), V_3 = (0, 1, 2, -1)$  and  $V_4 = (0, 1, -1, 0)$ . If V = (1, 2, -6, 2), compute [V]s. (10 Marks)

Module-2

3 a. Define an inner product space.

(03 Marks)

- b. For any vectors  $\alpha$ ,  $\beta$  in an inner product space V and any scalar C, prove that
  - (i)  $||c\alpha|| = c||\alpha||$

- (ii)  $\|\alpha\| > 0$  for  $\alpha \neq 0$
- (iii)  $||(\alpha|\beta)|| \le ||\alpha|| + ||\beta||$
- (iv)  $||\alpha + \beta|| \le ||\alpha|| + ||\beta||$

(07 Marks)

c. Construct an orthogonal basis for w given

$$\mathbf{x}_{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{x}_{2} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \qquad \mathbf{x}_{3} = \begin{bmatrix} 5 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$

(10 Marks)

OR

4 a. Define orthogonal projection.

(03 Marks)

- b. Prove that an orthogonal set of non-zero vectors is linearly independent and hence forms the basis for the subspace spanned by the set. (07 Marks)
- c. Find the least square solution of the system Ax = b for  $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ ,  $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$  and also

determine the least square error in the solution of Ax = b.

(10 Marks)

Module-3

- 5 a. Diagonalize the matrix  $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ , find  $A^6$ . (10 Marks)
  - b. Convert the quadratic form  $Q(x) = x_1^2 8x_1x_2 5x_2^2$  into quadratic form with no cross product. (10 Marks)

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## 20SCN/SCE/SSE/SCS/SIT/SIS/SFC/SAM/SAD/SDS11

a. Find the maximum and minimum values of  $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$  subject to the constraint (10 Marks)  $X^TX = 1$ .

b. Find the singular value decomposition of  $A = \begin{bmatrix} 1 & 1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$ (10 Marks)

a. Find the correlation coefficient and the line of regression of y on x for the

X	1	2	3	4	5
У	2	5	3	8	7

(10 Marks)

b. Fit a straight line for the following data:

X	1	2	3	4	5	6	7
У	9	8	10	12	11	13	14

(10 Marks)

a. Show that  $\tan \theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$  where  $\theta$  is acute angle. Explain the significance of the

formula when r = 0 and  $r = \pm 1$ .

(10 Marks)

b. Fit a parabola  $y = a + bx + cx^2$  for the following data:

X	1	2	3	4
У	1.7	1.8	2.3	3.2

(10 Marks)

(10 Marks)

a. A random variable X has the following pmf for various values of X.

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X	0	1	2	3	4	5	6	7	8	
F(X)	K	3K	5K	7K	9K	11K	13K	15K	17K	

Solve: (i) Value of K

(ii) 
$$P(X < 3)$$
,  $P(X \ge 3)$ ,  $P(0 < X <= 5)$ 

(iii) Find cumulative distribution function

(iv) What is the smallest value of X for which  $P(X \le x) > 0.5$ ?

b. A certain injection administered to each of 12 patients resulted in the following increases of B.P: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the injection will be, in general, accompanied by an increase in BP? (10 Marks)

OR

10 a. A random variable X has the following pdf:

$$P(X) = \begin{cases} Ke^{-x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Solve: (i) Value of constant K (ii) Mean (iii) Variance (iv) F(0.5)

(10 Marks)

b. The following data show defective articles produced by 4 machines:

Machine	A	В	С	D
Production time	1	1	2	3
No. of defective	12	30	63	98

Do the figures indicate a significant difference in the performance of the machines?

[Use  $\chi^2_{0.05}$  ( $\gamma = 3$ ) = 7.815]

(10 Marks)