NEW SCHEME

05EC046

Reg. No. 4NMLEC0509

First Semester M.Tech. Degree Examination, January/February 2006

LDE/LBI/LDC/LEC

Linear Algebra

Time: 3 hrs.) (Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. **(a)** Given,

$$A = \begin{pmatrix} 1 & 2 & -2 & -4 & 1 \\ 2 & 4 & -3 & -6 & 1 \\ 3 & 6 & -3 & -6 & 1 \\ 4 & 8 & -4 & -8 & 1 \\ 5 & 10 & -12 & -24 & 8 \end{pmatrix}$$

Find the following :

i) The row reduced echelon from of A.

ii) The row rank and nullity of A.

iii) Find the general solution of the system $Ax = 0$.

iv) Find a basis for the row space of A. **(15 Marks)**

(b) If W_1 and W_2 are subspaces of a vector space V over a field F show that $W_1 + W_2$ is also a subspace of V. **(5 Marks)**

2. **(a)** Find the LU decomposition with $l_{ii} = 1$ for the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 4 & 5 & 4 & -1 \\ -4 & 1 & 4 & 4 \\ 6 & -3 & 3 & 1 \end{pmatrix}$$

(10 Marks)

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(b) Show that the polynomials of degree at most 3 with real coefficients is a vector space over the field of real numbers. **(10 Marks)**

3. **(a)** Let $T : C^2 \to C^2$ be defined as $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 + 4x_2 \\ x_1 - x_2 \end{pmatrix}$

Let $B_1 = \left\{ v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\};\ B_2 = \left\{ e_1 = \begin{pmatrix} 1 \\ i \end{pmatrix};\ e_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \right\}$

where $i = \sqrt{-1}$. Answer the following :

i) Is T a linear operator on C^2?

ii) Find $[T]_{B_1}$ and $[T]_{B_2}$

iii) What is the relation between $[T]_{B_1}$ and $[T]_{B_2}$? **(12 Marks)**

(b) Let $T: V \to W$ be a linear transformation and $B = \{v_1, v_2, \ldots, v_k\}$ be a basis fo.

i) If T is one-one show that $Tv_1, Tv_2, \ldots Tv_k$ is a linearly independent set in W.

ii) If T is onto show that $Tv_1, Tv_2, \ldots, Tv_k$ spans W. (3 Marks)

4. (a) Let

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{pmatrix}$$

Answer the following questions :

i) Find the characteristic and minimal polynomials of A.

ii) Is A diagonalizable?

iii) Find projections E_1 and E_2 such $E_1 + E_2 = I$, $\lambda_1 E_1 + \lambda_2 E_2 = A$, $E_1 E_2 = 0_{3\times 3} = E_2 E_1$, where λ_1 and λ_2 are the eigen values of A. (15 Marks)

(b) Let V be an n-dimensional vector space over C. Let $T: V \to V$ be a linear operator. Prove that if T is both diagonalizable and nilpotent, then $T = Z$, the zero operator. (5 Marks)

5. (a) Let $T: C^3 \to C^3$ be defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4x_1 - 2x_2 + 2x_3 \\ -x_1 + 3x_2 + x_3 \\ x_1 - x_2 + 5x_3 \end{pmatrix}$$

$$If\ W_1 = \left\{ x = \begin{pmatrix} \alpha \\ \alpha \\ 0 \end{pmatrix} ;\ \alpha \in C \right\},\ W_2 = \left\{ y = \begin{pmatrix} \beta \\ \gamma \\ \beta + \gamma \end{pmatrix} ;\ \beta, \gamma \in C \right\},$$

show that W_1 and W_2 give a T-invariant direct sum decomposition of C^3. (10 Marks)

(b) Given that

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -0 & 0 & 0 \end{pmatrix}$$

is a nilpotent matrix, determine a matrix P such that $P^{-1}AP$ is in Jordan canonic form. (10 Marks)

6. (a) Given the linear operator $T: C^4 \to C^4$ defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 + 6x_2 + 2x_4 \\ 2x_1 + 6x_2 - 2x_3 + 2x_4 \\ -2x_1 + 2x_2 + 4x_3 \\ 6x_1 - 6x_2 - 6x_3 + 8x_4 \end{pmatrix}$$

If the characteristic polynomial is given by $c(\lambda) = (\lambda - 8)^2(\lambda - 2)^2$, and that the eigen spaces of $\lambda_1 = 8$ and $\lambda_2 = 2$ respectively are

$$W_1 = \left\{ x = \begin{pmatrix} \alpha \\ \alpha \\ 0 \\ 0 \end{pmatrix} ;\ \alpha \in C \right\},\ W_2 = \left\{ y = \begin{pmatrix} \beta \\ 0 \\ \beta \\ 0 \end{pmatrix} ;\ \beta \in C \right\}$$

Find :

I) The Jordan from of T.

II) An ordered basis B for C^4 such that $[T]_B$ is in Jordan form.

III) A matrix P such that $P^{-1}[T]_B P$ is in Jordan canonical form. **(15 Marks)**

(b) If A is a 7×7 nilpotent matrix with minimal polynomial λ^4, what are the possibilities for the Jordan canonical form? **(5 Marks)**

7. (a) Given

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}, \; b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix}$$

determine

I) a QR factorizaton of A and hence

II) the least-squares solution of $Ax = b$. **(12 Marks)**

(b) Let A be a real $m \times n$ matrix. Show that the Null spaces of A and $A^T A$ are equal, where A^T denotes the transpose of the matrix A. **(8 Marks)**

8. (a) Find the maximum value of a function $Q(x_1, x_2) = 5x_1^2 + 5x_2^2 - 2x_1x_2$, subject to the constraint $x_1^2 + x_2^2 = 1$. Determine a vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ for which the maximum is attained. **(6 Marks)**

(b) Find a singular value decomposition of

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}.$$

(14 Marks)

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