CBCS SCHEME

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18AI56

Fifth Semester B.E. Degree Examination, Dec.2024/Jan.2025
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define vector spaces, subspaces. Show that the set S = {(1, 0, 1), (1, 1, 0), (–1, 0, –1) is linearly dependant in $V_3(R)$. **(10 Marks)**

b. By Gaussian elimination, find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$ **(10 Marks)**

OR

2 a. Solve the system of linear equations using elementary row operations.

$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20.$$ **(06 Marks)**

b. Define :

i) Norm on vector space U

ii) Inner product in R^n

iii) Angle between vectors. **(06 Marks)**

c. Find rank of the matrix A by reducing into echelon form

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$ **(08 Marks)**

Module-2

3 a. Write a note on orthogonal and orthonormal vectors. If $a = [-2, 1]^T$, $b = [-3, 1]^T$, $c = \left[\frac{4}{3}, -1, \frac{2}{3}\right]^T$ and $d = [5, 6, -1]^T$, then compute :

i) $\left(\frac{a.b}{a.a}\right).a$

ii) Find a unit vector 'u' in the direction c

iii) Show that 'd' is orthogonal to c. **(10 Marks)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **(10 Marks)**

OR

4 a. Find the values of determinant and trace of the matrix :

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & 4 \\ -8 & 0 & -1 \end{bmatrix}.$$ **(05 Marks)**

b. Compute the singular value decomposition of a matrix :

$$A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}.$$ **(15 Marks)**

Module-3

5 a. Compute the derivative of the function $h(x) = g[f(x)]$ where $g[f(x)] = [f(x)]^4$ and $f(x) = (2x + 1)$. **(06 Marks)**

b. Define gradient of the function $f(x_1, x_2)$ and hence find gradient of $f(x_1, x_2) = x_1^2 x_2 + x_1 x_2^3$. **(06 Marks)**

c. With used notations write the identities which are used in computing gradients of :

i) $[f(x)]^T$ ii) $tr[f(x)]$ iii) $det[f(x)]$ iv) $[f(x)]^{-1}$, with respect to the variable x. **(08 Marks)**

OR

6 a. Starting from definition, find the derivative of x^n. **(08 Marks)**

b. Obtain the Maclaurin's series of sin x + cos x, hence draw the graphs of $f(x) = f(0)$,

$$f(x) = f'(0) + \frac{x}{1!} f(0) \text{ and } f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0).$$ **(12 Marks)**

Module-4

7 a. State and prove Baye's theorem on conditional probability. **(08 Marks)**

b. Let A and B be two events, which are not mutually exclusive and are connected with random experiment. Given that $P(A) = 3/4$ $P(B) = 1/5$ $P(A \cap B) = 1/20$ then find: i) $P(A \cup B)$ ii) $P(A \cap \overline{B})$ iii) $P(\overline{A} \cap B)$ iv) $P(A/B)$ and $P(B/A)$. **(06 Marks)**

c. A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find : i) Value of K ii) $P(x < 6)$ iii) $P(x \geq 6)$. **(06 Marks)**

OR

8 a. Test whether the following function is a density function $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ if so determine the probability that the variate having its density function will fall in the interval (1, 2). **(08 Marks)**

b. The length of the telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth i) Ends in less than 5 minutes ii) Between 5 and 10 minutes. **(06 Marks)**

c. Define binomial distribution and find the binomial probability distribution which has mean 2 and variance 4/3. **(06 Marks)**

Module-5

9 a. Find the maximum of $Z = 2x + 3y$
subject to the constraints $x + y \leq 30$,
$y \geq 30$,
$0 \leq y \geq 12$,
$x - y \geq 0$
and $0 \leq x \leq 20$. **(10 Marks)**

b. For convex functions f(y) and g(x), show that
$\min_{x} f(Ax) + g(x) = \min_{u} - f^*(u) - g^*(-A^T u)$. **(10 Marks)**

OR

10 a. Consider the linear program given below and derive the dual linear program using Lagrage duality.

$$\min_{x \in R^2} \frac{1}{2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Subject to $\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. **(10 Marks)**

b. Discuss the optimization using gradient descent, conjugate gradient, subgradient methods. Differential the methods if any. **(10 Marks)**

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