Second Semester M.Tech. Degree Examination, June/July 2019 **Error Control Coding**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

Define mutual information. Derive and explain the properties of mutual information.

A discrete memory less source has alphabet of five symbols with their probabilities as given below:

Syml	ool	S ₀	S_1	S ₂	S ₃	S ₄
Proba	abilities	0.4	0.2	0.2	0.1	0.1

Compute Huffman code by placing composite symbol as high as possible and by placing composite symbol as low as possible. Also find the:

i) The average code word length

ii) The variance of the average code word for both of the cases.

(10 Marks)

a. For a set integers $G = \{0, 1, 2, --- m - 1\}$ where m is nay +ve integer, show that

 $(i \boxplus j) \boxplus k = i \boxplus (j \boxplus k)$ where \boxplus denotes module in addition.

(08 Marks)

b. Explain the construction of Galois field G^F(2^m) a field F.

(08 Marks)

c. Let V be a vector space over a field F.

Prove that for any C in F and any V in $V(-C)\cdot(V) = C\cdot(-V) = -(C\cdot V)$.

(04 Marks)

Module-2

The syndrome of a (7, 4) linear code is given by

$$S_0 = r_0 + r_3 + r_5 + r_6$$

$$S_1 = r_1 + r_3 + r_4 + r_5$$

$$S_2 = r_2 + r_4 + r_5 + r_6$$

Find the following:

- i) Find the generator matrix draw the encoder circuit
- ii) Draw syndrome circuit
- iii) Find all possible code vectors
- iv) How many errors can it detect and correct?
- v) Detect and correct errors if r = 1001010.

(14 Marks)

If C = DG is a valid code vector prove that $CH^{T} = 0$ where H^{T} is transpose of parity check matrix H. (06 Marks)

OR

Write a note on product codes and interleaved codes.

- Form the generator matrix of a second order reed Muller code RM (r = 2, m = 4) of length 16. What is the minimum distance of the code? (10 Marks)

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Module-3

A (15, 5) binary cyclic code has a generator polynomial $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$.

i) Draw the encoder block diagram

- ii) Find the code polynomial for message polynomial $d(x) = 1 + x^2 + x^4$ in systematic form.
- iii) Is $v(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ is a code polynomial? If not find the syndrome of v(x).

b. With a block diagram, explain the decoding operation of error trapping decoder for a(15, 7) cyclic code generated by $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. (10 Marks)

With a block diagram explain decoding circuit for (31, 26) cyclic Hamming code generated by $g(x) = 1 + x^2 + x^5$. If the above Hamming code is shortened by three digits. Draw and explain the decoding circuit for resultant (28, 23) shortened cyclic code. (20 Marks)

Module-4

- Give the circuit for Galois field $GF(2^4)$ adder and multiplier (for multiplying $GF(2^4)$ by α^3) and explain their operation. What is requirement of these circuits?
 - b. Give the important parameters and features of RS code. Give the encoding circuit for q-ray R - S code and explain the symbols used in the circuit. (10 Marks)

- Example with suitable diagram type-I one step majority logic decoder error correction procedure. (10 Marks)
 - b. Draw and explain type-2, two step majority logic $g(x) = 1 + x + x^3$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$
 (10 Marks)

- a. For a rate $\frac{1}{2}$ convolutional encoder with a transfer function:
 - $G(x) = [1 + x^2 + x^3, 1 + x + x^2 + x^3]$, draw the encoder circuit and state diagram, hence evaluate the codeword produced by the input sequence 1 0 1 1 1. (10 Marks) (10 Marks)
 - b. With a flow chart explain ZJ or stack algorithm.

a. Explain the steps involved in viterbi algorithm. 10

(06 Marks)

b. Consider the convolutional encoder with $g(x) = [1 + x, 1 + x^2, 1 + x + x^2]$. If the received sequence v = [110, 110, 110, 111, 010, 101, 101]. Using Viterbi algorithm find the transmitted bit sequence. Assume that the codeword is transmitted over BSC channel.

(14 Marks)