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Fifth Semester B.E. Degree Examination, June/July 2024 Mathematic for Machine Learning

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find all solutions of the inhomogeneous system of linear equations $Ax = b$ where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(07 Marks)

- b. Find the image and kernel of a linear. Mapping

$$\phi: \mathbb{R}^4 \rightarrow \mathbb{R}^2, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(07 Marks)

- c. Consider \mathbb{R}^3 with $\langle \cdot, \cdot \rangle$ defined for all $x, y \in \mathbb{R}^3$ as $\langle x, y \rangle = x^T A y$, $A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 4 & -1 \\ 1 & -1 & 5 \end{bmatrix}$
Is $\langle \cdot, \cdot \rangle$ an inner product? (06 Marks)

OR

- 2 a. Find all solutions of system of equations :

$$-2x_1 + 4x_2 - 2x_3 - x_4 + 4x_5 = -3$$

$$4x_1 - 8x_2 + 3x_3 - 3x_4 + x_5 = 2$$

$$x_1 - 2x_2 + x_3 - x_4 + x_5 = 0$$

$$x_1 - 2x_2 - 3x_4 + 4x_5 = a.$$

(07 Marks)

- b. Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for \mathbb{R}^3 .
Express each of the standard basis vector Q linear combination of $\alpha_1, \alpha_2, \alpha_3$. (07 Marks)

- c. Define an inner product space. For any vector α, β in an inner product space V prove that $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$. (06 Marks)

Module-2

- 3 a. For a subspace $U = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\} \leq \mathbb{R}^3$ and $x = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^3$ find the coordinates λ of x in terms of the subspace U , the projection point $\pi_U(x)$ and the projection matrix P_π . (10 Marks)

- b. Diagonalize the matrix $A = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 3 & -2 \\ 1 & 2 & -1 \end{bmatrix}$. (10 Marks)

OR

- 4 a. Apply gram Schmidt orthogonalization process to the basis $B = (1, 1, 1), (-1, 0, 1), (-1, 2, 3)$ of the inner product space \mathbb{R}^3 to find an orthogonal basis of \mathbb{R}^3 . Also find orthogonal basis of \mathbb{R}^3 . (10 Marks)

- b. Find singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. (10 Marks)

Module-3

- 5 a. Compute the partial derivative $\frac{\partial f}{\partial A}$ for the function $f = Ax$ where $A \in \mathbb{R}^{3 \times 2}$ and $x \in \mathbb{R}^2$. (07 Marks)
- b. Consider $f(x_1, x_2) = x_1^2 + 2x_2$ where $x_1 = \sin t$ and $x_2 = \cos t$. find derivative of f with respect to t . (06 Marks)
- c. Obtain the gradient $\frac{df}{dx}$ for the function $f(x) = Ax$, $f(x) \in \mathbb{R}^M$, $A \in \mathbb{R}^{M \times N}$, $x \in \mathbb{R}^N$. (07 Marks)

OR

- 6 a. Consider the linear model $y = \phi^T \theta$ where $\theta \in \mathbb{R}^D$ is a parameter vector, $\phi \in \mathbb{R}^{N \times D}$ are input features and $y \in \mathbb{R}^N$ are corresponding observation we define least squares loss function :
 $L(\theta) = \frac{1}{2} \|y - \phi\theta\|^2$, $e(\theta) = y - \phi\theta$. Find $\frac{\partial L}{\partial \theta}$. (06 Marks)
- b. For the function $f(x) = \sqrt{x^2 + \exp(x^2) + \cos(x^2 + \exp(x^2))}$ find $\frac{\partial f}{\partial x}$. (07 Marks)
- c. Consider the matrix $R \in \mathbb{R}^{M \times N}$ and $f : \mathbb{R}^{M \times N} \rightarrow \mathbb{R}^{N \times N}$ with $f(R) = R^T R = K \in \mathbb{R}^{N \times N}$ find gradient dK/dR . (07 Marks)

Module-4

- 7 a. The probability that the noise level of a wide band amplifier will exceed 2dB is 0.05. Find the probabilities that among 12 such amplifiers the noise level of :
 i) One will exceed 2dB
 ii) Atmost 2 will exceed 2dB
 iii) Two or more will exceed 2dB. (06 Marks)
- b. Let X_1 and X_2 have the joint probability distribution :

$X_1 \backslash X_2$	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

- i) Find marginal distribution of x_1 and x_2
 ii) Find $P(x_1 + x_2 > 1)$
 iii) Find conditional probability distribution of x_1 given $x_2 = 1$. And x_1 and x_2 are Independent. (07 Marks)
- c. If x is a Poisson variate such that $P(x = 2) = 9P(x = 4) + 90P(x = 6)$. Find mean of x . (07 Marks)