CBCS SCHEME

USN [| | | | | | | | |] **18EC44**

Fourth Semester B.E. Degree Examination, June/July 2024
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive mean, variance and characteristic function for uniformly distributed random variable. **(10 Marks)**

b. The cdf for random variable z is $F_z(z) = \begin{cases} 1-\exp(-2z^{3/2}) & z \geq 0 \\ 0 & \text{otherwise} \end{cases}$.

Evaluate $P(0.5 < z \leq 0.9)$ **(04 Marks)**

c. It is given that $E[X] = 2$ and $E[X^2] = 6$.
(i) Find standard deviation of X.
(ii) If $Y = 6X^2 + 2X - 13$. Find mean of Y. **(06 Marks)**

OR

2 a. Given the data in the following table:

K	1	2	3	4	5
x_K	2.1	3.2	4.8	5.4	6.9
$p(x_K)$	0.21	0.18	0.20	0.22	0.19

(i) Plot pdf and cdf of discrete random variable X.
(ii) Write expression for $f_x(x)$ and $F_x(x)$ using unit delta functions and unit step functions. **(08 Marks)**

b. The random variable X is uniformly distributed between 0 and 2. $Y = 3x^3$. What is the pdf of X? **(06 Marks)**

c. The ransom variable X is uniformly distributed between 0 and 5. The event B is $B = \{X > 3.7\}$. What are $f_{X/B^{(x)}}$, $\mu_{X/B}$ and $\sigma^2_{X/B}$? **(06 Marks)**

Module-2

3 a. A bivariate Pdf is given as
$f_{XY}(x, y) = 0.2\delta(x)\,\delta(y) + 0.3\delta(x-1)\,\delta(y) + 0.3\delta(x)\,\delta(y-1) + C\delta(x-1)\,\delta(y-1)$
i) What is the value of the constant C?
ii) What are the Pdfs for X and Y?
iii) What is $F_{XY}(x, y)$ when $(0 < x < 1)$ and $(0 < y < 1)$?
iv) What are $F_{XY}(x, \alpha)$ and $F_{XY}(\alpha, y)$
v) Are X and Y independent? **(08 Marks)**

b. The mean and variance of random variable X are -1 and 2. The mean and variance of random variable Y are 3 and 4. The correlation coefficient $\rho_{XY} = 0.5$. What are the covariance COV[XY] and the correlation E[XY]. **(05 Marks)**

c. Write a short note on Chi-square random variable and students random variable. **(07 Marks)**

OR

4 a. X is a random variable, $\mu_X = 4$ and $\sigma_X = 5$, Y is a random variable, $\mu_Y = 6$ and $\sigma_Y = 7$. The correlation coefficient is 0.2. If $U = 3X + 2Y$. What are var[u], cov[uX] and cov[uY]? **(08 Marks)**

Let 'X' and 'Y' be exponentially distributed random variable with $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

b. Obtain the characteristic function and Pdf of $W = X + Y$. **(06 Marks)**

c. The Random variables X_i have same mean of $m_x = 4$ and variance of $\sigma_X^2 = 1.5$. For $w = \sum_{i=1}^{150} X_i$, determine m_w and σ_w^2. Also for $w = \frac{1}{150} \sum_{i=1}^{150} X_i$, determine m_y and $\sigma^2 y$. Comment on the result. **(06 Marks)**

Module-3

5 a. Define the following:
   (i) Random processes
   (ii) Stationary processes. **(04 Marks)**

b. Write the properties of Autocorrelation function. **(06 Marks)**

c. Show that the random process $X(t) = A\cos(\omega_C t + \theta)$ is wide sense stationary. 'θ' is uniformly distributed in the range $-\pi$ to π. **(10 Marks)**

OR

6 a. For the random process $X(t) = A\cos(\omega_C t + \theta)$, A and ω_C are constants. θ is a random variable, uniformly distributed between $\pm\pi$. Show that this process is ergodic. **(08 Marks)**

b. Determine the power spectral density of the random process $X(t) = A\cos(\omega_C t + \theta)$ and plot the same. Here θ is random variable uniformly distributed over 0 to 2π. Hence obtain average power of X(t). If the frequency becomes zero, X(t) = A i.e. a d.c. signal, then obtain power spectral density and autocorrelation function. **(08 Marks)**

c. A wide sense stationary random process X(t) is applied to a LTI system with impulse response $h(t) = ae^{-at}u(t)$. Find the mean value of the output Y(t) of the system if $E[X(t)] = 6$ and 'a' = 2. **(04 Marks)**

Module-4

7 a. Write the complete solution as x_p + multiplies of s in the null space.

$$x + 3y + 3z = 1$$
$$2x + 6y + 9z = 5$$
$$-x - 3y + 3z = 5$$

(06 Marks)

b. Find bases for the four subspaces associated with $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$. **(04 Marks)**

c. Find orthogonal vector A, B and orthonormal vector q_1 q_2 from a, b using Gram Schmidt process. Factorize into A = QR. $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. **(10 Marks)**

OR

8 a. Reduce A to echlon form. Which combination of rows of A produce zero row? What is the left Null space?

$$A = \begin{bmatrix} 1 & 2 & b_1 \\ 3 & 4 & b_2 \\ 4 & 6 & b_3 \end{bmatrix}$$ **(04 Marks)**

b. Project the vector b onto the line through a. Check that e is perpendicular to a.

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ **(08 Marks)**

c. In order to fit best straight line through four points passing through b = 0, 8, 8, 20 at t = 0, 1, 3, 4. Set up and solve normal equations $A^T A\hat{x} = A^T b$. **(08 Marks)**

Module-5

9 a. Mention the properties of determinants. **(08 Marks)**

b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, show that matrix A is positive definite matrix. **(06 Marks)**

c. Find the eigen values of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. **(06 Marks)**

OR

10 a. Diagonalize the following matrix, if possible $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. **(10 Marks)**

b. Find a singular value of decomposition of, $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. **(10 Marks)**

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